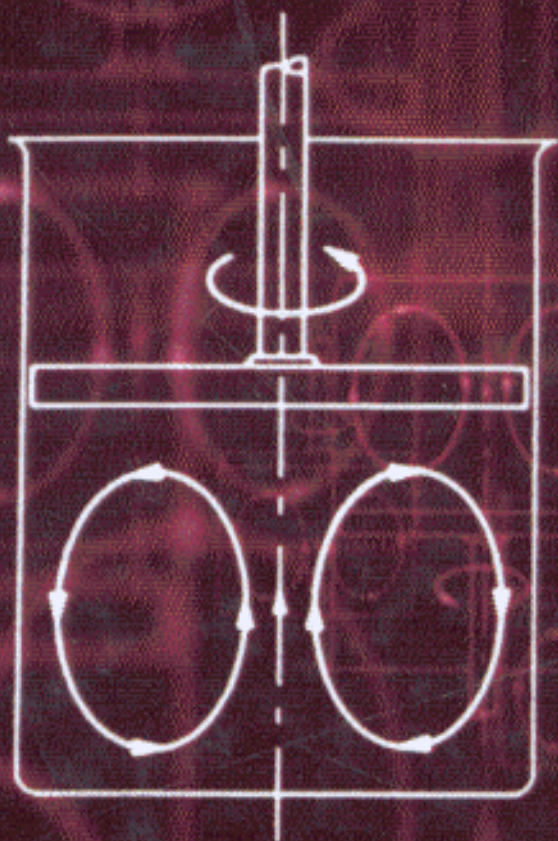


OXFORD ENGINEERING SCIENCE SERIES • 52

# ENGINEERING RHEOLOGY

SECOND EDITION

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