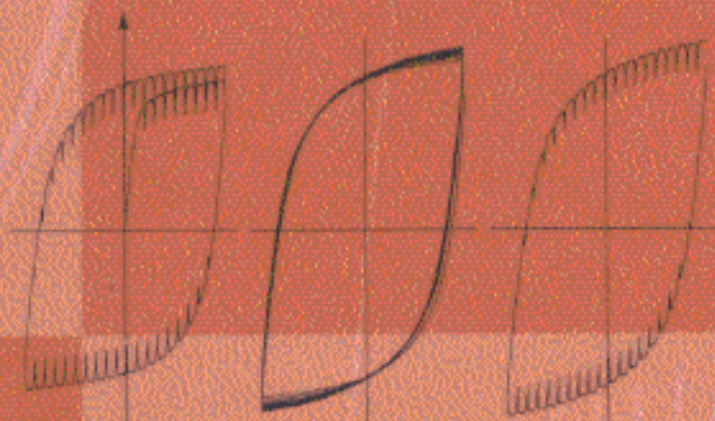


P. Haupt

Continuum Mechanics and Theory of Materials

Second Edition



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