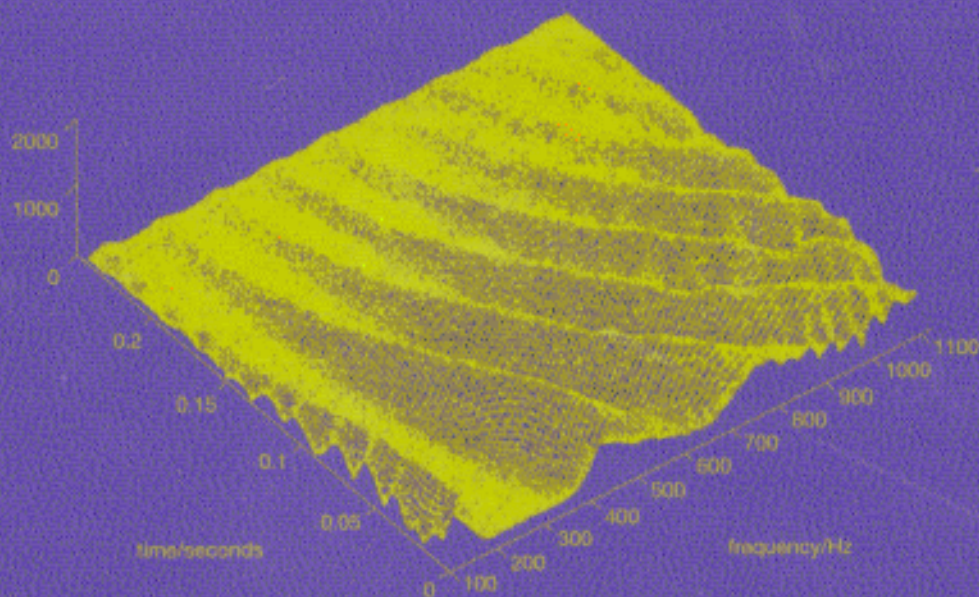


Foundations of Time-Frequency Analysis



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Birkhäuser

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