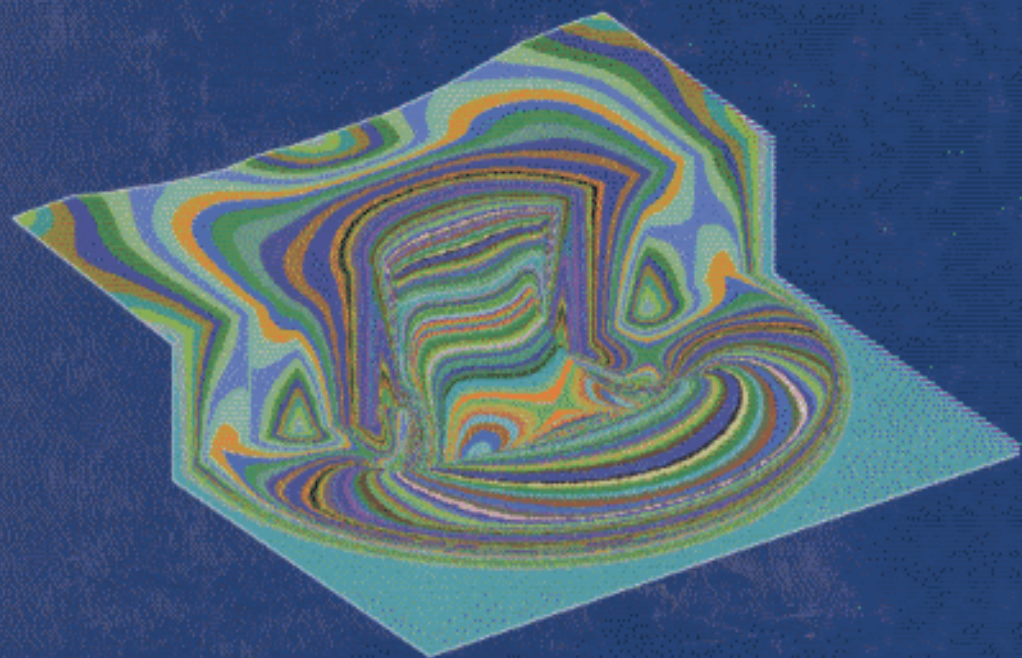


 WILEY

Shock-Capturing Methods for Free-Surface Shallow Flows



Eleuterio F. Toro

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