



Includes CD-ROM

# Boundary Element Programming in Mechanics

Xiao-Wei Gao  
Trevor G. Davies

# Contents

<i>Preface</i>	<i>page</i> xiii
<i>Legal Matters</i>	xv
<b>PART I. LINEAR PROBLEMS</b>	
<b>1 Introduction</b>	<b>3</b>
1.1. Introduction	3
1.2. A Note on Programming	3
1.3. Mathematical Preliminaries	5
1.4. Historical Sketch	12
1.4.1. Approximate Methods	12
1.4.2. BEM in Solid Mechanics	13
1.4.3. BEM in Elasticity	14
1.4.4. BEM in Elasto–Plasticity	14
1.5. Closure	15
<b>2 Theory of Elasticity</b>	<b>16</b>
2.1. Introduction	16
2.2. Displacements	16
2.3. Stresses	17
2.4. Stress–Strain Relationships	18
2.5. Navier–Cauchy Equations of Equilibrium	19
2.6. Reduced Forms in Two Dimensions	20
2.6.1. Plane Strain	21
2.6.2. Plane Stress	22
2.6.3. Axisymmetry	22
2.7. Anisotropic Materials	23
2.8. Closure	24
<b>3 Boundary Integral Equations for Elasticity</b>	<b>25</b>
3.1. Introduction	25
3.2. The Kelvin Fundamental Solution	25
3.3. Betti's Reciprocal Work Theorem	27
3.4. Somigliana Identity	28
3.5. Boundary Integral Equations	30

3.6. Internal Stresses	31
3.7. Closure	33
<b>4 Numerical Implementation</b>	<b>34</b>
4.1. Introduction	34
4.2. Boundary Discretization	34
4.3. Interpolation of Field Quantities	37
4.4. Discretized Boundary Integral Equations	38
4.5. Adaptive Integration	39
4.6. Singular Integration	47
4.6.1. Weakly Singular Integrals in Three Dimensions	47
4.6.2. Weakly Singular Integrals in Two Dimensions	49
4.6.3. Strongly Singular Integrals	51
4.7. Evaluation of Boundary Stresses	53
4.8. Symmetry	57
4.9. Corners and Edges	60
4.10. Multiple Regions	63
4.11. System Equation Solution	64
4.12. Closure	66
<b>5 The Elastic Program Code</b>	<b>67</b>
5.1. Introduction	67
5.2. Scope of the Program	67
5.3. Program Structure	67
5.4. Global Variables	68
5.4.1. Global Variables in Module Program Units	68
5.4.2. Global Variables Passed through Argument Lists	75
5.5. Main Program: BEMECH	75
5.6. Subroutine INPUT_CTR	76
5.7. Subroutine BLOCK_DATA	77
5.8. Subroutine INPUT_EL	78
5.9. Subroutine TREAT_T	79
5.10. Subroutine AXES_COS	80
5.11. Subroutine SHAPEF	81
5.12. Subroutines DSHAPE and DSHAP3D	82
5.13. Subroutine EL_COEFS	84
5.14. Subroutine ADAPTINT	85
5.15. Subroutine CHOSEGP	87
5.16. Subroutines SETGAS and GAUSSV	89
5.17. Subroutines MINDIST and IVSNR123	90
5.18. Subroutine INT_HG	91
5.19. Subroutine EVAL_HG	92
5.20. Subroutine FORM_HG	93
5.21. Subroutine SINGUHG	94
5.22. Subroutine SIN2DHG	95
5.23. Subroutine SETDSUB	96
5.24. Subroutine BDSTRS	97
5.25. Subroutine BSCOEF	98
5.26. Subroutine SYMTRY	99
5.27. Subroutine HGTOEQS	100

5.28. Subroutine INNERPS	101
5.29. Subroutine EL_SOLVE	102
5.30. Subroutine INVSOLVR	103
5.31. Subroutines OUTPUT and SIGTITL	104
5.32. Closure	106
<b>6 Linear Applications</b>	<b>107</b>
6.1. Introduction	107
6.2. Thick-Walled Cylinder under Internal Pressure	107
6.3. Circular Rigid Foundation on a Semi-Infinite Medium	111
6.4. A Three-Dimensional Machine Component	112
6.5. Closure	117
<b>PART II. NONLINEAR PROBLEMS</b>	
<b>7 Rate-Independent Plasticity Theory</b>	<b>121</b>
7.1. Introduction	121
7.2. Isotropic Yield Criteria	121
7.2.1. Stress Invariants and Principal Stresses	122
7.2.2. The Tresca Criterion	123
7.2.3. The Von Mises Criterion	124
7.2.4. The Mohr-Coulomb Criterion	124
7.2.5. The Drucker-Prager Criterion	125
7.3. Principles of Elasto-Plastic Flow	126
7.4. Constitutive Relationships	128
7.5. Isotropic Hardening Materials	130
7.5.1. Equivalent Uniaxial Yield Criteria	131
7.5.2. Equivalent Plastic Strain	131
7.5.3. Explicit Derivations	132
7.6. Kinematic Hardening Materials	134
7.7. Mixed Hardening Materials	135
7.8. Closure	136
<b>8 Boundary Integral Equations in Elasto-Plasticity</b>	<b>138</b>
8.1. Introduction	138
8.2. Boundary Integral Equations	138
8.3. Internal Stress Integral Equations	140
8.4. Integration of Strongly Singular Domain Integrals	143
8.5. Closure	145
<b>9 Numerical Implementation</b>	<b>146</b>
9.1. Introduction	146
9.2. Domain Discretization	146
9.3. Weakly Singular Domain Integrals	148
9.4. Strongly Singular Domain Integrals	149
9.5. Boundary Stresses – Traction-Recovery Method	150
9.6. System Equations	153
9.6.1. Initial Stress Representation	153
9.6.2. Plastic Multiplier Representation	154
9.7. System Equation Solution	156
9.7.1. Newton-Raphson Method	156

9.7.2. Transition from Elastic to Elasto–Plastic States	157
9.7.3. Automatic Incrementation of Boundary Loading	158
9.7.4. Summary	159
9.8. Closure	160
<b>10 The Elasto–Plastic Program Code</b>	162
10.1. Introduction	162
10.2. Scope of the Program	162
10.3. Program Structure	162
10.4. Global Variables	164
10.5. Subroutine INPUT_NL	165
10.6. Subroutine NL_COEFS	166
10.7. Subroutine INT_CELL	167
10.8. Subroutine SIN_CELL	168
10.9. Subroutine CELL_BOUND	170
10.10. Subroutine EVAL_KE	171
10.11. Subroutine INTSUBC	173
10.12. Subroutine PBSCOEF	173
10.13. Subroutine NL_SOLVE	174
10.14. Subroutine PL_FLOW	176
10.15. Subroutine DF_DSIG	177
10.16. Subroutine P_M_ITER	178
10.17. Subroutine MATRICES	181
10.18. Subroutine SIG_SCALE	182
10.19. Subroutine SIGCROSS	183
10.20. Subroutine DF_MATRX	184
10.21. Subroutine UPDATEV	185
10.22. Closure	186
<b>11 Nonlinear Applications</b>	187
11.1. Introduction	187
11.2. A Cube Subjected to Uniaxial Tension	187
11.3. A Thick-Walled Cylinder Subjected to Internal Pressure	191
11.4. A Rigid Punch under Plane Strain	193
11.5. A Flexible Square Footing	196
11.6. Multiplanar Tubular DX-Joint	199
11.7. Closure	201
<b>12 Epilogue</b>	203
12.1. Review	203
12.2. The Way Forward	203
12.2.1. Automatic Integration	204
12.2.2. Computation of Boundary Stresses	204
12.2.3. Stress-Return Algorithm	204
12.2.4. System Equation Solver	205
12.2.5. Local Boundary Conditions	205
12.2.6. Nonlinear Hardening	206
12.2.7. Advanced Yield Functions	206
12.2.8. Finite Strain Elasto–Plasticity	206
12.2.9. Infinite Boundary Elements	207
12.2.10. Multiple Regions	207

<b>Appendix A. Derivation of Kernel Functions</b>	209
A.1. Derivation of the Strain Kernel	209
A.2. Derivation of the Stress Kernel	210
A.3. Derivation of the Traction Kernel	211
A.4. Kernel Functions for Plane Strain and Plane Stress	211
<b>Appendix B. Shape Functions</b>	213
B.1. One-Dimensional Shape Functions	213
B.2. Two-Dimensional Shape Functions	214
B.3. Three-Dimensional Shape Functions	215
<b>Appendix C. Degenerate Elements: Singular Mapping</b>	217
<b>Appendix D. Elasto–Plastic Flow Theory</b>	221
D.1. Derivation of the Plastic Flow Rule and Plastic Loading Rule	221
D.2. Derivations for Kinematic Hardening Materials	224
D.3. Derivations for Mixed Hardening Materials	225
D.4. Derivation of the Deformation State Function $\Gamma$	226
<b>Appendix E. Domain Integral Formulations</b>	228
E.1. Boundary Integral Equations: Initial Strain Formulation	228
E.2. Analytical Integration of the Strongly Singular Volume Integral	228
E.3. <i>Interior Stress Equation: Initial Strain Formulation</i>	230
E.4. Analytical Integration of $E_{ijkl}$ in Two Dimensions	230
E.5. Analytical Integration: Initial Strain Formulation	231
<b>Appendix F. Solution of the Nonlinear System Equations</b>	232
F.1. The Newton–Raphson Iterative Algorithm	232
F.2. System Equation Solution Strategies	233
F.2.1. The Initial Stress Iteration Technique	234
F.2.2. The Implicit Solution Technique	235
F.2.3. The Variable Stiffness Technique	236
F.2.4. The Mixed Representation Technique	237
<b>Appendix G. Elements of Elasto–Plasticity</b>	239
<b>Appendix H. Description of Input Data</b>	242
<i>References</i>	247
<i>Index</i>	253