

Mathematical Modeling in Continuum Mechanics

Roger Temam
Alain Miranville

Contents

<i>Introduction</i>	page ix
<i>A Few Words About Notations</i>	xii

PART ONE. FUNDAMENTAL CONCEPTS IN CONTINUUM MECHANICS

1 Describing the Motion of a System: Geometry and Kinematics	3
1.1 Deformations	3
1.2 Motion and Its Observation (Kinematics)	6
1.3 Description of the Motion of a System: Eulerian and Lagrangian Derivatives	10
1.4 Velocity Field of a Rigid Body: Helicoidal Vector Fields	12
1.5 Differentiation of a Volume Integral Depending on a Parameter	17
2 The Fundamental Law of Dynamics	21
2.1 The Concept of Mass	21
2.2 Forces	27
2.3 The Fundamental Law of Dynamics and Its First Consequences	28
2.4 Application to Systems of Material Points and to Rigid Bodies	31
2.5 Galilean Frames: The Fundamental Law of Dynamics Expressed in a Non-Galilean Frame	35
3 The Cauchy Stress Tensor – Applications	38
3.1 Hypotheses on the Cohesion Forces	38
3.2 The Cauchy Stress Tensor	41
3.3 General Equations of Motion	44
3.4 Symmetry of the Stress Tensor	46

4	Real and Virtual Powers	48
4.1	Study of a System of Material Points	48
4.2	General Material Systems: Rigidifying Velocities	52
4.3	Virtual Power of the Cohesion Forces: The General Case	54
4.4	Real Power: The Kinetic Energy Theorem	58
5	Deformation Tensor, Deformation Rate Tensor, Constitutive Laws	60
5.1	Further Properties of Deformations	60
5.2	The Deformation Rate Tensor	65
5.3	Introduction to Rheology: The Constitutive Laws	67
6	Energy Equations and Shock Equations	77
6.1	Heat and Energy	77
6.2	Shocks and the Rankine–Hugoniot Relations	82
PART TWO. PHYSICS OF FLUIDS		
7	General Properties of Newtonian Fluids	89
7.1	General Equations of Fluid Mechanics	89
7.2	Statics of Fluids	95
7.3	Remark on the Energy of a Fluid	100
8	Flows of Inviscid Fluids	102
8.1	General Theorems	102
8.2	Plane Irrotational Flows	106
8.3	Transsonic Flows	116
8.4	Linear Acoustics	120
9	Viscous Fluids and Thermohydraulics	122
9.1	Equations of Viscous Incompressible Fluids	122
9.2	Simple Flows of Viscous Incompressible Fluids	123
9.3	Thermohydraulics	129
9.4	Equations in Nondimensional Form: Similarities	131
9.5	Notions of Stability and Turbulence	133
9.6	Notion of Boundary Layer	137
10	Magnetohydrodynamics and Inertial Confinement of Plasmas	141
10.1	The Maxwell Equations and Electromagnetism	142
10.2	Magnetohydrodynamics	146
10.3	The Tokamak Machine	148
11	Combustion	153
11.1	Equations for Mixtures of Fluids	153
11.2	Equations of Chemical Kinetics	155

11.3	The Equations of Combustion	157
11.4	Stefan–Maxwell Equations	159
11.5	A Simplified Problem: The Two-Species Model	162
12	Equations of the Atmosphere and of the Ocean	164
12.1	Preliminaries	165
12.2	Primitive Equations of the Atmosphere	167
12.3	Primitive Equations of the Ocean	171
12.4	Chemistry of the Atmosphere and the Ocean	172
	Appendix: The Differential Operators in Spherical Coordinates	174
PART THREE. SOLID MECHANICS		
13	The General Equations of Linear Elasticity	179
13.1	Back to the Stress–Strain Law of Linear Elasticity: The Elasticity Coefficients of a Material	179
13.2	Boundary Value Problems in Linear Elasticity: The Linearization Principle	181
13.3	Other Equations	186
13.4	The Limit of Elasticity Criteria	189
14	Classical Problems of Elastostatics	191
14.1	Longitudinal Traction–Compression of a Cylindrical Bar	191
14.2	Uniform Compression of an Arbitrary Body	194
14.3	Equilibrium of a Spherical Container Subjected to External and Internal Pressures	195
14.4	Deformation of a Vertical Cylindrical Body Under the Action of Its Weight	198
14.5	Simple Bending of a Cylindrical Beam	201
14.6	Torsion of Cylindrical Shafts	205
14.7	The Saint-Venant Principle	208
15	Energy Theorems – Duality: Variational Formulations	210
15.1	Elastic Energy of a Material	210
15.2	Duality – Generalization	212
15.3	The Energy Theorems	215
15.4	Variational Formulations	218
15.5	Virtual Power Theorem and Variational Formulations	221
16	Introduction to Nonlinear Constitutive Laws and to Homogenization	223
16.1	Nonlinear Constitutive Laws (Nonlinear Elasticity)	224
16.2	Nonlinear Elasticity with a Threshold (Henky’s Elastoplastic Model)	226

16.3	Nonconvex Energy Functions	228
16.4	Composite Materials: The Problem of Homogenization	230
PART FOUR. INTRODUCTION TO WAVE PHENOMENA		
17	Linear Wave Equations in Mechanics	235
17.1	Returning to the Equations of Linear Acoustics and of Linear Elasticity	236
17.2	Solution of the One-Dimensional Wave Equation	239
17.3	Normal Modes	241
17.4	Solution of the Wave Equation	245
17.5	Superposition of Waves, Beats, and Packets of Waves	249
18	The Soliton Equation: The Korteweg–de Vries Equation	252
18.1	Water-Wave Equations	253
18.2	Simplified Form of the Water-Wave Equations	255
18.3	The Korteweg–de Vries Equation	258
18.4	The Soliton Solutions of the KdV Equation	262
19	The Nonlinear Schrödinger Equation	264
19.1	Maxwell Equations for Polarized Media	265
19.2	Equations of the Electric Field: The Linear Case	267
19.3	General Case	270
19.4	The Nonlinear Schrödinger Equation	274
19.5	Soliton Solutions of the NLS Equation	277
Appendix	The Partial Differential Equations of Mechanics	279
	<i>References</i>	281
	<i>Index</i>	285