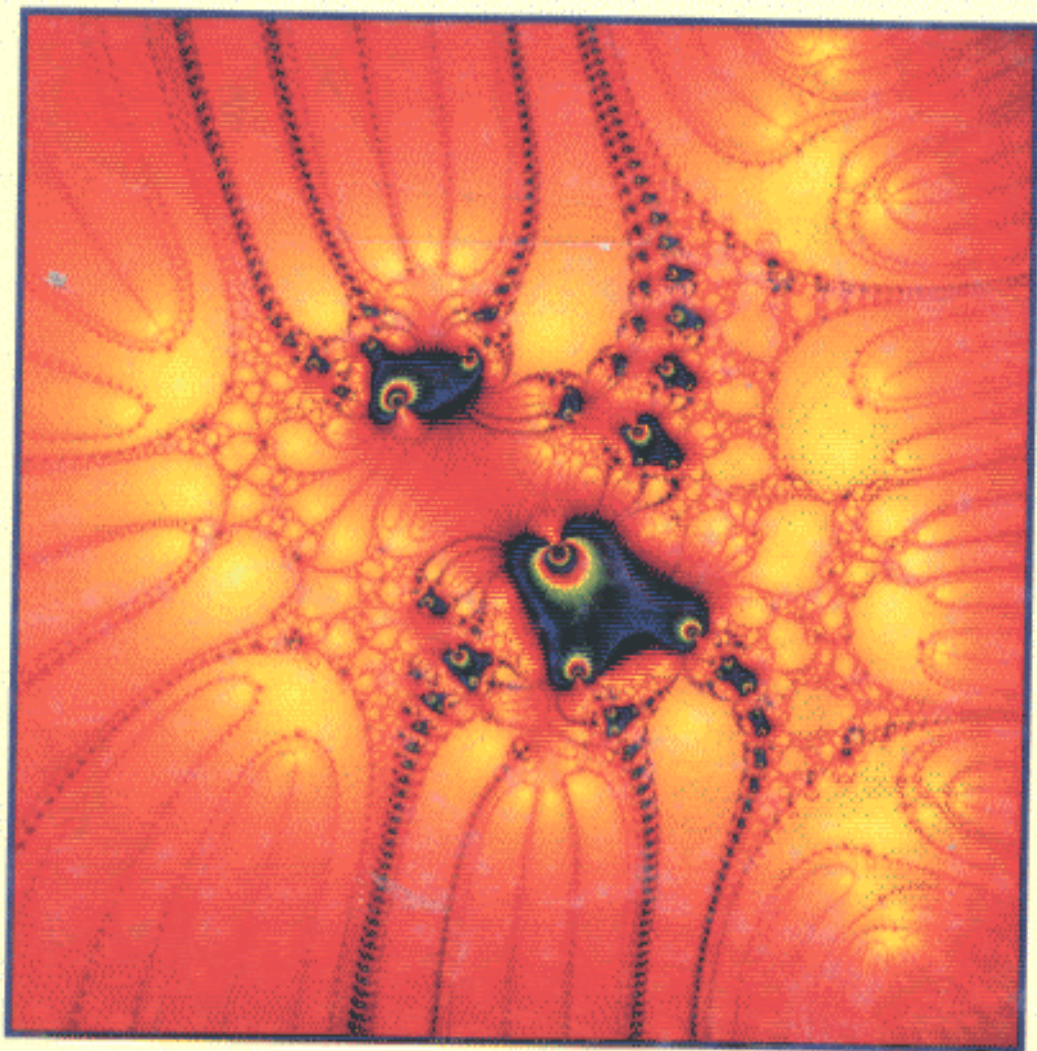


Vector Calculus, Linear Algebra, and Differential Forms

A Unified Approach

Second Edition



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