

ARTHUR P. BORESI
RICHARD J. SCHMIDT

ADVANCED MECHANICS OF MATERIALS



Sixth Edition

CHAPTER 1 INTRODUCTION 1

- 1.1 Review of Elementary Mechanics of Materials 1
 - 1.1.1 Axially Loaded Members 1
 - 1.1.2 Torsionally Loaded Members 3
 - 1.1.3 Bending of Beams 3
- 1.2 Methods of Analysis 5
 - 1.2.1 Method of Mechanics of Materials 6
 - 1.2.2 Method of Continuum Mechanics and the Theory of Elasticity 7
 - 1.2.3 Deflections by Energy Methods 7
- 1.3 Stress-Strain Relations 8
 - 1.3.1 Elastic and Inelastic Response of a Solid 8
 - 1.3.2 Material Properties 10
- 1.4 Failure and Limits on Design 16
 - 1.4.1 Modes of Failure 19

CHAPTER 2 THEORIES OF STRESS AND STRAIN 25

- 2.1 Definition of Stress at a Point 25
- 2.2 Stress Notation 26
- 2.3 Symmetry of the Stress Array and Stress on an Arbitrarily Oriented Plane 28
 - 2.3.1 Symmetry of Stress Components 28
 - 2.3.2 Stresses Acting on Arbitrary Planes 29
 - 2.3.3 Normal Stress and Shear Stress on an Oblique Plane 30
- 2.4 Transformation of Stress, Principal Stresses, and Other Properties 31
 - 2.4.1 Transformation of Stress 31
 - 2.4.2 Principal Stresses 32
 - 2.4.3 Principal Values and Directions 33
 - 2.4.4 Octahedral Stress 36
 - 2.4.5 Mean and Deviator Stresses 37
 - 2.4.6 Plane Stress 38
 - 2.4.7 Mohr's Circle in Two Dimensions 40
 - 2.4.8 Mohr's Circles in Three Dimensions 43
- 2.5 Differential Equations of Motion of a Deformable Body 50
 - 2.5.1 Specialization of Equations 2.46 52
- 2.6 Deformation of a Deformable Body 54
- 2.7 Strain Theory, Transformation of Strain, and Principal Strains 55
 - 2.7.1 Strain of a Line Element 55

2.7.2 Final Direction of a Line Element 57

2.7.3 Rotation Between Two Line Elements (Definition of Shear Strain) 58

2.7.4 Principal Strains 60

2.8 Small-Displacement Theory 61

2.8.1 Strain Compatibility Relations 62

2.8.2 Strain-Displacement Relations for Orthogonal Curvilinear Coordinates 63

2.9 Strain Measurement and Strain Rosettes 70

CHAPTER 3 LINEAR STRESS-STRAIN-TEMPERATURE RELATIONS 79

- 3.1 First Law of Thermodynamics, Internal-Energy Density, and Complementary Internal-Energy Density 79
 - 3.1.1 Elasticity and Internal-Energy Density 81
 - 3.1.2 Elasticity and Complementary Internal-Energy Density 82
- 3.2 Hooke's Law: Anisotropic Elasticity 84
- 3.3 Hooke's Law: Isotropic Elasticity 85
 - 3.3.1 Isotropic and Homogeneous Materials 85
 - 3.3.2 Strain-Energy Density of Isotropic Elastic Materials 85
- 3.4 Equations of Thermoelasticity for Isotropic Materials 91
- 3.5 Hooke's Law: Orthotropic Materials 93

CHAPTER 4 INELASTIC MATERIAL BEHAVIOR 104

- 4.1 Limitations on the Use of Uniaxial Stress-Strain Data 104
 - 4.1.1 Rate of Loading 105
 - 4.1.2 Temperature Lower Than Room Temperature 105
 - 4.1.3 Temperature Higher Than Room Temperature 105
 - 4.1.4 Unloading and Load Reversal 105
 - 4.1.5 Multiaxial States of Stress 106
- 4.2 Nonlinear Material Response 107
 - 4.2.1 Models of Uniaxial Stress-Strain Curves 108
- 4.3 Yield Criteria: General Concepts 113
 - 4.3.1 Maximum Principal Stress Criterion 114
 - 4.3.2 Maximum Principal Strain Criterion 116
 - 4.3.3 Strain-Energy Density Criterion 116

X CONTENTS

4.4	Yielding of Ductile Metals	117	6.6	Torsion of Rectangular Cross Section Members	222
4.4.1	Maximum Shear-Stress (Tresca) Criterion	118	6.7	Hollow Thin-Wall Torsion Members and Multiply Connected Cross Sections	228
4.4.2	Distortional Energy Density (von Mises) Criterion	120	6.7.1	Hollow Thin-Wall Torsion Member Having Several Compartments	230
4.4.3	Effect of Hydrostatic Stress and the π -Plane	122	6.8	Thin-Wall Torsion Members with Restrained Ends	234
4.5	Alternative Yield Criteria	126	6.8.1	I-Section Torsion Member Having One End Restrained from Warping	235
4.5.1	Mohr–Coulomb Yield Criterion	126	6.8.2	Various Loads and Supports for Beams in Torsion	239
4.5.2	Drucker–Prager Yield Criterion	128	6.9	Numerical Solution of the Torsion Problem	239
4.5.3	Hill's Criterion for Orthotropic Materials	128	6.10	Inelastic Torsion: Circular Cross Sections	243
4.6	General Yielding	129	6.10.1	Modulus of Rupture in Torsion	244
4.6.1	Elastic–Plastic Bending	131	6.10.2	Elastic–Plastic and Fully Plastic Torsion	244
4.6.2	Fully Plastic Moment	132	6.10.3	Residual Shear Stress	246
4.6.3	Shear Effect on Inelastic Bending	134	6.11	Fully Plastic Torsion: General Cross Sections	250
4.6.4	Modulus of Rupture	134			
4.6.5	Comparison of Failure Criteria	136			
4.6.6	Interpretation of Failure Criteria for General Yielding	137			
CHAPTER 5 APPLICATIONS OF ENERGY METHODS 147			CHAPTER 7 BENDING OF STRAIGHT BEAMS 263		
5.1	Principle of Stationary Potential Energy	147	7.1	Fundamentals of Beam Bending	263
5.2	Castigliano's Theorem on Deflections	152	7.1.1	Centroidal Coordinate Axes	263
5.3	Castigliano's Theorem on Deflections for Linear Load–Deflection Relations	155	7.1.2	Shear Loading of a Beam and Shear Center Defined	264
5.3.1	Strain Energy U_N for Axial Loading	156	7.1.3	Symmetrical Bending	265
5.3.2	Strain Energies U_M and U_S for Beams	158	7.1.4	Nonsymmetrical Bending	268
5.3.3	Strain Energy U_T for Torsion	160	7.1.5	Plane of Loads: Symmetrical and Nonsymmetrical Loading	268
5.4	Deflections of Statically Determinate Structures	163	7.2	Bending Stresses in Beams Subjected to Nonsymmetrical Bending	272
5.4.1	Curved Beams Treated as Straight Beams	165	7.2.1	Equations of Equilibrium	272
5.4.2	Dummy Load Method and Dummy Unit Load Method	170	7.2.2	Geometry of Deformation	273
5.5	Statically Indeterminate Structures	177	7.2.3	Stress–Strain Relations	273
5.5.1	Deflections of Statically Indeterminate Structures	180	7.2.4	Load–Stress Relation for Nonsymmetrical Bending	273
CHAPTER 6 TORSION 200			7.2.5	Neutral Axis	274
6.1	Torsion of a Prismatic Bar of Circular Cross Section	200	7.2.6	More Convenient Form for the Flexure Stress σ_{zz}	275
6.1.1	Design of Transmission Shafts	204	7.3	Deflections of Straight Beams Subjected to Nonsymmetrical Bending	280
6.2	Saint-Venant's Semiinverse Method	209	7.4	Effect of Inclined Loads	284
6.2.1	Geometry of Deformation	209	7.5	Fully Plastic Load for Nonsymmetrical Bending	285
6.2.2	Stresses at a Point and Equations of Equilibrium	210			
6.2.3	Boundary Conditions	211	CHAPTER 8 SHEAR CENTER FOR THIN-WALL BEAM CROSS SECTIONS 295		
6.3	Linear Elastic Solution	213	8.1	Approximations for Shear in Thin-Wall Beam Cross Sections	295
6.3.1	Elliptical Cross Section	214	8.2	Shear Flow in Thin-Wall Beam Cross Sections	296
6.3.2	Equilateral Triangle Cross Section	215	8.3	Shear Center for a Channel Section	298
6.3.3	Other Cross Sections	216	8.4	Shear Center of Composite Beams Formed from Stringers and Thin Webs	303
6.4	The Prandtl Elastic-Membrane (Soap-Film) Analogy	216	8.5	Shear Center of Box Beams	306
6.4.1	Remark on Reentrant Corners	219			
6.5	Narrow Rectangular Cross Section	219			
6.5.1	Cross Sections Made Up of Long Narrow Rectangles	221			

CHAPTER 9 CURVED BEAMS 319

- 9.1 Introduction 319
- 9.2 Circumferential Stresses in a Curved Beam 320
 - 9.2.1 Location of Neutral Axis of Cross Section 326
- 9.3 Radial Stresses in Curved Beams 333
 - 9.3.1 Curved Beams Made from Anisotropic Materials 334
- 9.4 Correction of Circumferential Stresses in Curved Beams Having I, T, or Similar Cross Sections 338
 - 9.4.1 Bleich's Correction Factors 340
- 9.5 Deflections of Curved Beams 343
 - 9.5.1 Cross Sections in the Form of an I, T, etc. 346
- 9.6 Statically Indeterminate Curved Beams: Closed Ring Subjected to a Concentrated Load 348
- 9.7 Fully Plastic Loads for Curved Beams 350
 - 9.7.1 Fully Plastic Versus Maximum Elastic Loads for Curved Beams 351

CHAPTER 10 BEAMS ON ELASTIC FOUNDATIONS 357

- 10.1 General Theory 357
- 10.2 Infinite Beam Subjected to a Concentrated Load: Boundary Conditions 360
 - 10.2.1 Method of Superposition 363
 - 10.2.2 Beam Supported on Equally Spaced Discrete Elastic Supports 364
- 10.3 Infinite Beam Subjected to a Distributed Load Segment 369
 - 10.3.1 Uniformly Distributed Load 369
 - 10.3.2 $\beta L' \leq \pi$ 371
 - 10.3.3 $\beta L' \rightarrow \infty$ 371
 - 10.3.4 Intermediate Values of $\beta L'$ 371
 - 10.3.5 Triangular Load 371
- 10.4 Semiinfinite Beam Subjected to Loads at Its End 374
- 10.5 Semiinfinite Beam with Concentrated Load Near Its End 376
- 10.6 Short Beams 377
- 10.7 Thin-Wall Circular Cylinders 378

CHAPTER 11 THE THICK-WALL CYLINDER 389

- 11.1 Basic Relations 389
 - 11.1.1 Equation of Equilibrium 391
 - 11.1.2 Strain–Displacement Relations and Compatibility Condition 391
 - 11.1.3 Stress–Strain–Temperature Relations 392
 - 11.1.4 Material Response Data 392
- 11.2 Stress Components at Sections Far from Ends for a Cylinder with Closed Ends 392
 - 11.2.1 Open Cylinder 394
- 11.3 Stress Components and Radial Displacement for Constant Temperature 395
 - 11.3.1 Stress Components 395

- 11.3.2 Radial Displacement for a Closed Cylinder 396
- 11.3.3 Radial Displacement for an Open Cylinder 396
- 11.4 Criteria of Failure 399
 - 11.4.1 Failure of Brittle Materials 399
 - 11.4.2 Failure of Ductile Materials 400
 - 11.4.3 Material Response Data for Design 400
 - 11.4.4 Ideal Residual Stress Distributions for Composite Open Cylinders 401
- 11.5 Fully Plastic Pressure and Autofrettage 405
- 11.6 Cylinder Solution for Temperature Change Only 409
 - 11.6.1 Steady-State Temperature Change (Distribution) 409
 - 11.6.2 Stress Components 410
- 11.7 Rotating Disks of Constant Thickness 411

CHAPTER 12 ELASTIC AND INELASTIC STABILITY OF COLUMNS 423

- 12.1 Introduction to the Concept of Column Buckling 424
- 12.2 Deflection Response of Columns to Compressive Loads 425
 - 12.2.1 Elastic Buckling of an Ideal Slender Column 425
 - 12.2.2 Imperfect Slender Columns 427
- 12.3 The Euler Formula for Columns with Pinned Ends 428
 - 12.3.1 The Equilibrium Method 428
 - 12.3.2 Higher Buckling Loads; $n > 1$ 431
 - 12.3.3 The Imperfection Method 432
 - 12.3.4 The Energy Method 433
- 12.4 Euler Buckling of Columns with Linearly Elastic End Constraints 436
- 12.5 Local Buckling of Columns 440
- 12.6 Inelastic Buckling of Columns 442
 - 12.6.1 Inelastic Buckling 442
 - 12.6.2 Two Formulas for Inelastic Buckling of an Ideal Column 443
 - 12.6.3 Tangent-Modulus Formula for an Inelastic Buckling Load 444
 - 12.6.4 Direct Tangent-Modulus Method 446

CHAPTER 13 FLAT PLATES 457

- 13.1 Introduction 457
- 13.2 Stress Resultants in a Flat Plate 458
- 13.3 Kinematics: Strain–Displacement Relations for Plates 461
 - 13.3.1 Rotation of a Plate Surface Element 464
- 13.4 Equilibrium Equations for Small-Displacement Theory of Flat Plates 466
- 13.5 Stress–Strain–Temperature Relations for Isotropic Elastic Plates 469

13.5.1	Stress Components in Terms of Tractions and Moments	472	14.3	Stress Concentration Factors: Combined Loads	515
13.5.2	Pure Bending of Plates	472	14.3.1	Infinite Plate with a Circular Hole	515
13.6	Strain Energy of a Plate	472	14.3.2	Elliptical Hole in an Infinite Plate Uniformly Stressed in Directions of Major and Minor Axes of the Hole	516
13.7	Boundary Conditions for Plates	473	14.3.3	Pure Shear Parallel to Major and Minor Axes of the Elliptical Hole	516
13.8	Solution of Rectangular Plate Problems	476	14.3.4	Elliptical Hole in an Infinite Plate with Different Loads in Two Perpendicular Directions	517
13.8.1	Solution of $\nabla^2 \nabla^2 w = \frac{p}{D}$ for a Rectangular Plate	477	14.3.5	Stress Concentration at a Groove in a Circular Shaft	520
13.8.2	Westergaard Approximate Solution for Rectangular Plates: Uniform Load	479	14.4	Stress Concentration Factors: Experimental Techniques	522
13.8.3	Deflection of a Rectangular Plate: Uniformly Distributed Load	482	14.4.1	Photoelastic Method	522
13.9	Solution of Circular Plate Problems	486	14.4.2	Strain-Gage Method	524
13.9.1	Solution of $\nabla^2 \nabla^2 w = \frac{p}{D}$ for a Circular Plate	486	14.4.3	Elastic Torsional Stress Concentration at a Fillet in a Shaft	525
13.9.2	Circular Plates with Simply Supported Edges	488	14.4.4	Elastic Membrane Method: Torsional Stress Concentration	525
13.9.3	Circular Plates with Fixed Edges	488	14.4.5	Beams with Rectangular Cross Sections	527
13.9.4	Circular Plate with a Circular Hole at the Center	489	14.5	Effective Stress Concentration Factors	530
13.9.5	Summary for Circular Plates with Simply Supported Edges	490	14.5.1	Definition of Effective Stress Concentration Factor	530
13.9.6	Summary for Circular Plates with Fixed Edges	491	14.5.2	Static Loads	532
13.9.7	Summary for Stresses and Deflections in Flat Circular Plates with Central Holes	492	14.5.3	Repeated Loads	532
13.9.8	Summary for Large Elastic Deflections of Circular Plates: Clamped Edge and Uniformly Distributed Load	492	14.5.4	Residual Stresses	534
13.9.9	Significant Stress When Edges Are Clamped	495	14.5.5	Very Abrupt Changes in Section: Stress Gradient	534
13.9.10	Load on a Plate When Edges Are Clamped	496	14.5.6	Significance of Stress Gradient	535
13.9.11	Summary for Large Elastic Deflections of Circular Plates: Simply Supported Edge and Uniformly Distributed Load	497	14.5.7	Impact or Energy Loading	536
13.9.12	Rectangular or Other Shaped Plates with Large Deflections	498	14.6	Effective Stress Concentration Factors: Inelastic Strains	536
			14.6.1	Neuber's Theorem	537
CHAPTER 14 STRESS CONCENTRATIONS 502			CHAPTER 15 FRACTURE MECHANICS 543		
14.1	Nature of a Stress Concentration Problem and the Stress Concentration Factor	504	15.1	Failure Criteria and Fracture	544
14.2	Stress Concentration Factors: Theory of Elasticity	507	15.1.1	Brittle Fracture of Members Free of Cracks and Flaws	545
14.2.1	Circular Hole in an Infinite Plate Under Uniaxial Tension	507	15.1.2	Brittle Fracture of Cracked or Flawed Members	545
14.2.2	Elliptic Hole in an Infinite Plate Stressed in a Direction Perpendicular to the Major Axis of the Hole	508	15.2	The Stationary Crack	551
14.2.3	Elliptical Hole in an Infinite Plate Stressed in the Direction Perpendicular to the Minor Axis of the Hole	511	15.2.1	Blunt Crack	553
14.2.4	Crack in a Plate	512	15.2.2	Sharp Crack	554
14.2.5	Ellipsoidal Cavity	512	15.3	Crack Propagation and the Stress Intensity Factor	555
14.2.6	Grooves and Holes	513	15.3.1	Elastic Stress at the Tip of a Sharp Crack	555
			15.3.2	Stress Intensity Factor: Definition and Derivation	556
			15.3.3	Derivation of Crack Extension Force G	556
			15.3.4	Critical Value of Crack Extension Force	558
			15.4	Fracture: Other Factors	561
			15.4.1	Elastic-Plastic Fracture Mechanics	562
			15.4.2	Crack-Growth Analysis	562

- 15.4.3 Load Spectra and Stress History **562**
- 15.4.4 Testing and Experimental Data Interpretation **563**

- 17.9.4 Maximum Octahedral Shear Stress **617**
- 17.9.5 Effect of Magnitude of Friction Coefficient **618**
- 17.9.6 Range of Shear Stress for One Load Cycle **619**

CHAPTER 16 FATIGUE: PROGRESSIVE FRACTURE 567

- 16.1 Fracture Resulting from Cyclic Loading **568**
 - 16.1.1 Stress Concentrations **573**
- 16.2 Effective Stress Concentration Factors: Repeated Loads **575**
- 16.3 Effective Stress Concentration Factors: Other Influences **575**
 - 16.3.1 Corrosion Fatigue **575**
 - 16.3.2 Effect of Range of Stress **577**
 - 16.3.3 Methods of Reducing Harmful Effects of Stress Concentrations **577**
- 16.4 Low Cycle Fatigue and the ϵ - N Relation **580**
 - 16.4.1 Hysteresis Loop **580**
 - 16.4.2 Fatigue-Life Curve and the ϵ - N Relation **581**

CHAPTER 17 CONTACT STRESSES 589

- 17.1 Introduction **589**
- 17.2 The Problem of Determining Contact Stresses **590**
- 17.3 Geometry of the Contact Surface **591**
 - 17.3.1 Fundamental Assumptions **591**
 - 17.3.2 Contact Surface Shape After Loading **592**
 - 17.3.3 Justification of Eq. 17.1 **592**
 - 17.3.4 Brief Discussion of the Solution **595**
- 17.4 Notation and Meaning of Terms **596**
- 17.5 Expressions for Principal Stresses **597**
- 17.6 Method of Computing Contact Stresses **598**
 - 17.6.1 Principal Stresses **598**
 - 17.6.2 Maximum Shear Stress **599**
 - 17.6.3 Maximum Octahedral Shear Stress **599**
 - 17.6.4 Maximum Orthogonal Shear Stress **599**
 - 17.6.5 Curves for Computing Stresses for Any Value of B/A **605**
- 17.7 Deflection of Bodies in Point Contact **607**
 - 17.7.1 Significance of Stresses **611**
- 17.8 Stress for Two Bodies in Line Contact: Loads Normal to Contact Area **611**
 - 17.8.1 Maximum Principal Stresses: $k = 0$ **613**
 - 17.8.2 Maximum Shear Stress: $k = 0$ **613**
 - 17.8.3 Maximum Octahedral Shear Stress: $k = 0$ **613**
- 17.9 Stresses for Two Bodies in Line Contact: Loads Normal and Tangent to Contact Area **613**
 - 17.9.1 Roller on Plane **614**
 - 17.9.2 Principal Stresses **616**
 - 17.9.3 Maximum Shear Stress **617**

CHAPTER 18 CREEP: TIME-DEPENDENT DEFORMATION 624

- 18.1 Definition of Creep and the Creep Curve **624**
- 18.2 The Tension Creep Test for Metals **626**
- 18.3 One-Dimensional Creep Formulas for Metals Subjected to Constant Stress and Elevated Temperature **626**
- 18.4 One-Dimensional Creep of Metals Subjected to Variable Stress and Temperature **631**
 - 18.4.1 Preliminary Concepts **631**
 - 18.4.2 Similarity of Creep Curves **633**
 - 18.4.3 Temperature Dependency **635**
 - 18.4.4 Variable Stress and Temperature **635**
- 18.5 Creep Under Multiaxial States of Stress **640**
 - 18.5.1 General Discussion **640**
- 18.6 Flow Rule for Creep of Metals Subjected to Multiaxial States of Stress **643**
 - 18.6.1 Steady-State Creep **644**
 - 18.6.2 Nonsteady Creep **648**
- 18.7 An Application of Creep of Metals **649**
 - 18.7.1 Summary **650**
- 18.8 Creep of Nonmetals **650**
 - 18.8.1 Asphalt **650**
 - 18.8.2 Concrete **651**
 - 18.8.3 Wood **652**

APPENDIX A AVERAGE MECHANICAL PROPERTIES OF SELECTED MATERIALS 657**APPENDIX B SECOND MOMENT (MOMENT OF INERTIA) OF A PLANE AREA 660**

- B.1 Moments of Inertia of a Plane Area **660**
- B.2 Parallel Axis Theorem **661**
- B.3 Transformation Equations for Moments and Products of Inertia **664**
 - B.3.1 Principal Axes of Inertia **665**

APPENDIX C PROPERTIES OF STEEL CROSS SECTIONS 668**AUTHOR INDEX 673****SUBJECT INDEX 676**