

*CD-ROM Included*

# COMPUTATIONAL METHODS FOR LARGE SPARSE POWER SYSTEMS ANALYSIS

## An Object Oriented Approach

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