

Australian Mathematical Society Lecture Series 18

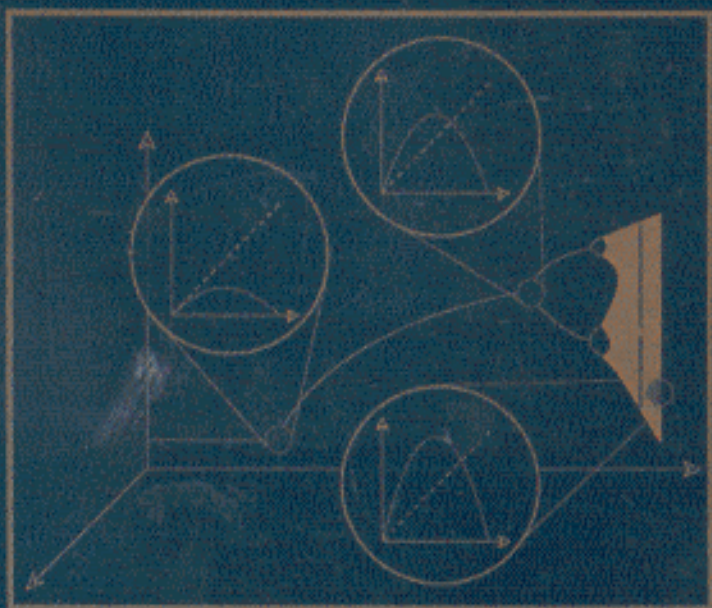
Chaos

A Mathematical Introduction

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