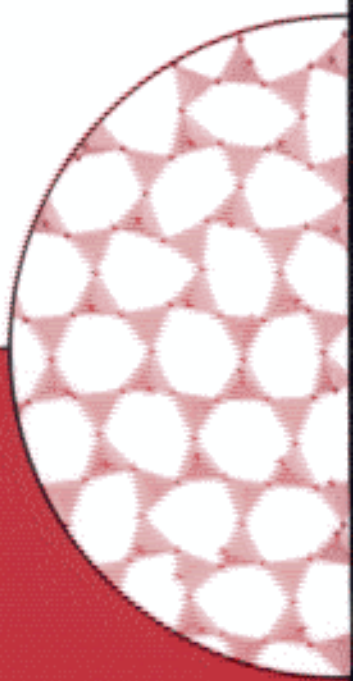
OXFORD MASTER SERIES IN CONDENSED MATTER PHYSICS

Structure and Dynamics

An atomic view of materials



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