

Cambridge studies in advanced mathematics

74

Real Analysis and Probability

R. M. DUDLEY

A decorative graphic consisting of several parallel orange diagonal stripes with white borders, running from the bottom left towards the top right of the cover.

Contents

<i>Preface to the Cambridge Edition</i>	<i>page ix</i>
1 Foundations; Set Theory	1
1.1 Definitions for Set Theory and the Real Number System	1
1.2 Relations and Orderings	9
*1.3 Transfinite Induction and Recursion	12
1.4 Cardinality	16
1.5 The Axiom of Choice and Its Equivalents	18
2 General Topology	24
2.1 Topologies, Metrics, and Continuity	24
2.2 Compactness and Product Topologies	34
2.3 Complete and Compact Metric Spaces	44
2.4 Some Metrics for Function Spaces	48
2.5 Completion and Completeness of Metric Spaces	58
*2.6 Extension of Continuous Functions	63
*2.7 Uniformities and Uniform Spaces	67
*2.8 Compactification	71
3 Measures	85
3.1 Introduction to Measures	85
3.2 Semirings and Rings	94
3.3 Completion of Measures	101
3.4 Lebesgue Measure and Nonmeasurable Sets	105
*3.5 Atomic and Nonatomic Measures	109
4 Integration	114
4.1 Simple Functions	114
*4.2 Measurability	123
4.3 Convergence Theorems for Integrals	130

4.4 Product Measures	134
*4.5 Daniell-Stone Integrals	142
5 L^p Spaces; Introduction to Functional Analysis	152
5.1 Inequalities for Integrals	152
5.2 Norms and Completeness of L^p	158
5.3 Hilbert Spaces	160
5.4 Orthonormal Sets and Bases	165
5.5 Linear Forms on Hilbert Spaces, Inclusions of L^p Spaces, and Relations Between Two Measures	173
5.6 Signed Measures	178
6 Convex Sets and Duality of Normed Spaces	188
6.1 Lipschitz, Continuous, and Bounded Functionals	188
6.2 Convex Sets and Their Separation	195
6.3 Convex Functions	203
*6.4 Duality of L^p Spaces	208
6.5 Uniform Boundedness and Closed Graphs	211
*6.6 The Brunn-Minkowski Inequality	215
7 Measure, Topology, and Differentiation	222
7.1 Baire and Borel σ -Algebras and Regularity of Measures	222
*7.2 Lebesgue's Differentiation Theorems	228
*7.3 The Regularity Extension	235
*7.4 The Dual of $C(K)$ and Fourier Series	239
*7.5 Almost Uniform Convergence and Lusin's Theorem	243
8 Introduction to Probability Theory	250
8.1 Basic Definitions	251
8.2 Infinite Products of Probability Spaces	255
8.3 Laws of Large Numbers	260
*8.4 Ergodic Theorems	267
9 Convergence of Laws and Central Limit Theorems	282
9.1 Distribution Functions and Densities	282
9.2 Convergence of Random Variables	287
9.3 Convergence of Laws	291
9.4 Characteristic Functions	298
9.5 Uniqueness of Characteristic Functions and a Central Limit Theorem	303
9.6 Triangular Arrays and Lindeberg's Theorem	315
9.7 Sums of Independent Real Random Variables	320

*9.8 The Lévy Continuity Theorem; Infinitely Divisible and Stable Laws	325
10 Conditional Expectations and Martingales	336
10.1 Conditional Expectations	336
10.2 Regular Conditional Probabilities and Jensen's Inequality	341
10.3 Martingales	353
10.4 Optional Stopping and Uniform Integrability	358
10.5 Convergence of Martingales and Submartingales	364
*10.6 Reversed Martingales and Submartingales	370
*10.7 Subadditive and Superadditive Ergodic Theorems	374
11 Convergence of Laws on Separable Metric Spaces	385
11.1 Laws and Their Convergence	385
11.2 Lipschitz Functions	390
11.3 Metrics for Convergence of Laws	393
11.4 Convergence of Empirical Measures	399
11.5 Tightness and Uniform Tightness	402
*11.6 Strassen's Theorem: Nearby Variables with Nearby Laws	406
*11.7 A Uniformity for Laws and Almost Surely Converging Realizations of Converging Laws	413
*11.8 Kantorovich-Rubinstein Theorems	420
*11.9 <i>U</i> -Statistics	426
12 Stochastic Processes	439
12.1 Existence of Processes and Brownian Motion	439
12.2 The Strong Markov Property of Brownian Motion	450
12.3 Reflection Principles, The Brownian Bridge, and Laws of Suprema	459
12.4 Laws of Brownian Motion at Markov Times: Skorohod Imbedding	469
12.5 Laws of the Iterated Logarithm	476
13 Measurability: Borel Isomorphism and Analytic Sets	487
*13.1 Borel Isomorphism	487
*13.2 Analytic Sets	493
Appendix A Axiomatic Set Theory	503
A.1 Mathematical Logic	503
A.2 Axioms for Set Theory	505

A.3 Ordinals and Cardinals	510
A.4 From Sets to Numbers	515
Appendix B Complex Numbers, Vector Spaces, and Taylor's Theorem with Remainder	521
Appendix C The Problem of Measure	526
Appendix D Rearranging Sums of Nonnegative Terms	528
Appendix E Pathologies of Compact Nonmetric Spaces	530
<i>Author Index</i>	541
<i>Subject Index</i>	546
<i>Notation Index</i>	554