THE NONLINEAR WORKBOOK

2nd Edition

Chaos, Fractals,

Cellular Automata,

Neural Networks,

Genetic Algorithms,

Gene Expression Programming,

Wavelets,

Fuzzy Logic

with C++, Java and SymbolicC++ Programs

WILLI-HANS STEEB

World Scientific

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