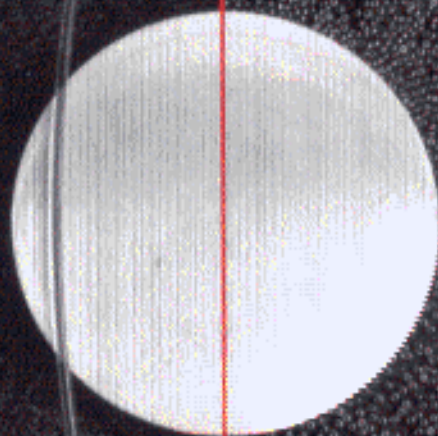


PROBABILITY,  
STATISTICS, *and*  
RANDOM PROCESSES  
*for* ENGINEERS



RICHARD H. WILLIAMS

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