

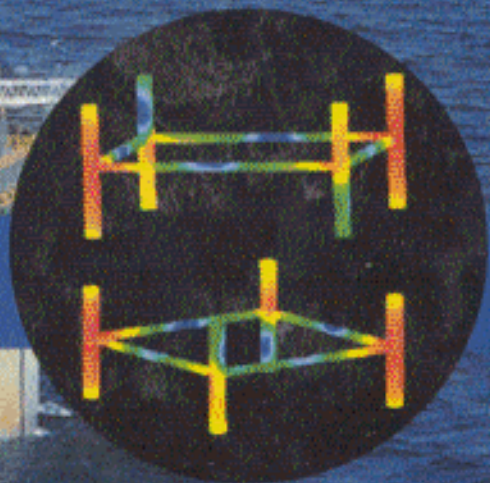
THE BOUNDARY ELEMENT METHOD

VOLUME 1

Applications in
Thermo-Fluids and Acoustics

L. C. Wrobel

 WILEY



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