

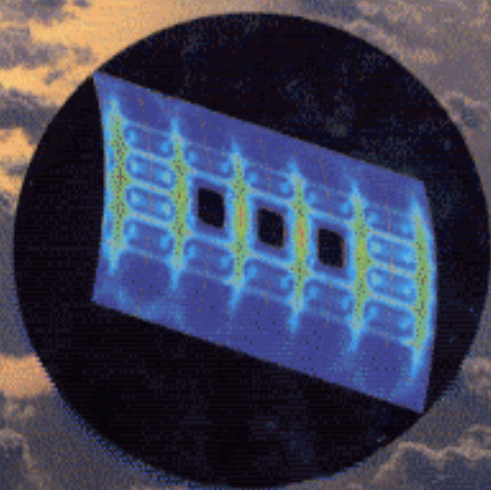
THE BOUNDARY ELEMENT METHOD

VOLUME 2

Applications in
Solids and Structures



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 WILEY

Contents

Preface	xiii
Preface to Volume 2	xv
Acknowledgements	xvii
1 Introduction	1
1.1 Applications	4
1.2 Mathematical Preliminaries	5
1.2.1 Cartesian Tensor Notation	5
1.2.2 Divergence Theorem	8
1.2.3 Dirac Delta Function	8
1.3 Overview of the Book	8
1.3.1 Volume 1	9
1.3.2 Volume 2	11
2 Elastostatics	15
2.1 Introduction	15
2.2 Basic Elasticity	16
2.2.1 Stress	16
2.2.2 Equilibrium	16
2.2.3 Plane Stress	19
2.2.4 Principal Stress	21
2.2.5 Strain	22
2.2.6 Compatibility Equations	23
2.2.7 Stress-Strain Relationship	24
2.2.8 Governing Equations of Elasticity	26
2.3 Boundary Element Formulations	26
2.3.1 Reciprocal Theorem	26
2.3.2 Betti's Reciprocal Theorem	28
2.3.3 Boundary Integral Equations	29
2.3.4 Fundamental Solutions	30
2.3.5 Boundary Displacement Equation	34
2.3.6 Axisymmetric Problems	37
2.3.7 Infinite Regions	38
2.3.8 Semi-infinite Regions	39
2.3.9 Boundary Stress Equation	40
2.3.10 Numerical Discretization	42
2.4 Stress and Displacements at Interior Points	56
2.5 Boundary Stresses	56

2.5.1	Indirect Approach	57
2.5.2	Direct Approach	61
2.6	Body Forces	61
2.6.1	Domain Discretization	62
2.6.2	Galerkin Vector	63
2.6.3	Particular Integrals	65
2.6.4	Multiple Reciprocity Method	67
2.6.5	Dual Reciprocity Method	67
2.7	Multi-region Formulation	70
2.8	Corner Problems	71
2.9	Anisotropic Elasticity	74
2.9.1	Basic Equations	74
2.9.2	Fundamental Solutions	75
2.10	Galerkin Formulation	76
2.10.1	Regularization	79
2.10.2	The Uniqueness Problem	80
2.11	Coupling Finite Elements and Boundary Elements	81
2.11.1	Stiffness Matrix Approach	81
2.11.2	Multi-region Approach	82
2.12	Examples	83
2.12.1	Pressurized Cylinder	83
2.12.2	Stress Concentration	87
2.12.3	Plate with an Elliptical Hole	87
2.12.4	A Strip Under Self-Weight and Centrifugal Loads	88
2.12.5	Orthotropic Circular Ring	90
2.12.6	Hole in a Strip	92
2.12.7	Analysis of Human Tibia	92
2.13	Summary	95
3	Plates and Shells	103
3.1	Introduction	103
3.2	Shear Deformable Plates	105
3.2.1	Basic Theory	105
3.2.2	Boundary Integral Formulations	109
3.2.3	Multi-region Formulation	113
3.2.4	Foundation Plates	114
3.3	Shear Deformable Shallow Shells	115
3.3.1	Basic Theory	115
3.3.2	Integral Equation Formulations	123
3.3.3	Boundary Integral Equations	124
3.3.4	Numerical Discretization	126
3.3.5	Stress Resultants Integral Equations	130
3.3.6	Hypersingular Integral Equations	131
3.3.7	Boundary Stress Resultants	133
3.4	Examples	134
3.4.1	Simply Supported Thin Square Plate	134
3.4.2	L-shaped Plate Structure	136
3.4.3	Cantilever Beam	136
3.4.4	X-core Structure	138

3.4.5	Shallow Spherical Shell	138
3.4.6	Simply Supported Spherical Shell	145
3.4.7	Cylindrical Shell	147
3.5	Kirchhoff Plate Theory	148
3.5.1	Basic Concepts	148
3.5.2	Boundary Integral Formulation	150
3.5.3	Example: Rectangular Plate with an Opening	151
3.6	Summary	154
4	Thermoelasticity	161
4.1	Introduction	161
4.2	Basic Concepts	162
4.2.1	Heat Conduction Equation	162
4.2.2	Thermoelasticity Equations	162
4.3	Boundary Integral Formulations	163
4.3.1	Steady State Thermoelasticity	163
4.3.2	Transient Thermoelasticity	174
4.4	Examples	186
4.4.1	Thick Hollow Cylinder	186
4.4.2	Transient Thermal Problems	187
4.5	Summary	191
5	Elastodynamics	195
5.1	Introduction	195
5.2	Transient Elastodynamics	196
5.2.1	Basic Concepts	196
5.2.2	Time Domain Integral Equations	196
5.2.3	Numerical Discretization	198
5.3	Laplace Transform Method	201
5.3.1	Basic Concepts	201
5.3.2	Integral Equation Formulations	201
5.3.3	Numerical Discretization	202
5.3.4	Laplace Inversion Method	203
5.4	Dual Reciprocity Method	204
5.4.1	Numerical Discretization	206
5.4.2	Dual Reciprocity in the Laplace Domain	207
5.4.3	Time Integration	208
5.5	Examples	209
5.5.1	A Strip Subjected to Pure Tension and Shear Loads	209
5.5.2	Plate with a Hole	211
5.5.3	Rectangular Bar Subjected to Tensile Load	211
5.5.4	Comparison of DRM, TDM and LTM	213
5.6	Acoustic Scattering in Fluid-Solid Problems	220
5.6.1	Example : Cylinder Immersed in Fluid	221
5.7	Summary	222

6	Elastoplasticity	229
6.1	Introduction	229
6.2	Basic Concepts	230
6.2.1	Elastoplastic Relationships	232
6.2.2	Multiaxial Loading	233
6.2.3	Plastic Stress-Strain Relationships	235
6.3	The Governing Elastoplastic Equations	237
6.4	Initial Strain Boundary Integral Formulation	239
6.4.1	Internal Stresses	241
6.4.2	Stresses at Boundary Points	243
6.4.3	Alternative Approaches	244
6.5	Numerical Discretization	245
6.5.1	Treatment of the Integrals	246
6.5.2	System Matrices Assembly	250
6.6	Non-linear Solution Algorithm	252
6.6.1	Implicit Procedures	252
6.6.2	Explicit Procedure	253
6.7	Examples	254
6.7.1	Two-dimensional Benchmarks	254
6.7.2	Three-dimensional Benchmarks	260
6.8	Summary	261
7	Contact Mechanics	269
7.1	Introduction	269
7.2	Basic Contact Mechanics	270
7.2.1	Friction	270
7.2.2	Classification of Contact	270
7.2.3	Modes of Contact	271
7.2.4	Local Coordinate System	272
7.2.5	Definition of Normal Gap	272
7.2.6	Numerical Modelling Concept	273
7.3	Elastostatic Contact	274
7.3.1	Numerical Modelling	277
7.3.2	Fully Incremental Loading Technique	279
7.3.3	Examples	284
7.4	Elastoplastic Contact	290
7.4.1	Incremental Formulation	291
7.4.2	Examples	292
7.5	Non-conforming Discretization Methods	301
7.5.1	Non-Conforming Discretization Formulations	301
7.5.2	Examples	306
7.6	Three-dimensional Contact problems	308
7.6.1	Contact Conditions	311
7.6.2	Example	311
7.7	Summary	313

8	Fracture Mechanics	319
8.1	Introduction	319
8.2	Basic Fracture Mechanics	320
8.2.1	Stress Intensity Factor	320
8.2.2	Modes of Fracture	320
8.2.3	Energy Balance	321
8.2.4	Stress and Displacements Fields	322
8.2.5	Residual Strength	324
8.2.6	Fatigue Crack Growth	325
8.2.7	Criteria for Crack Growth Direction	326
8.3	Modelling Co-planar Surfaces	328
8.3.1	Displacement Integral Equation	328
8.3.2	Traction Integral Equation	329
8.4	Dual Boundary Element Method	330
8.4.1	Crack Modelling Strategy	331
8.4.2	DBEM with Continuous Elements	341
8.5	Multi-region Method	343
8.6	Crack Green's Function Method	343
8.7	Displacement Discontinuity Method	345
8.8	Methods for Evaluating Stress Intensity Factors	346
8.8.1	Displacement Extrapolation Method	346
8.8.2	Subtraction of Singularity Method	357
8.8.3	Energy Release Rate Method	358
8.8.4	Path Independent Integrals	358
8.8.5	Energy Domain Integral	367
8.8.6	Weight Function Method	370
8.9	Examples	372
8.9.1	Edge Crack	372
8.9.2	Centre Crack	373
8.9.3	Centre Crack Plate Loaded by Tension and Bending	373
8.9.4	Double Edge Crack in a Composite Laminate	375
8.9.5	Penny-Shaped Crack	376
8.9.6	Inclined Penny-Shaped Crack	378
8.9.7	Centre Crack Panel Elastoplastic Analysis	378
8.9.8	Elliptical Crack Subjected to Impact Load	380
8.9.9	Interface Cracks	382
8.10	Crack Growth	383
8.10.1	Two-dimensional Modelling	384
8.10.2	Three-dimensional Modelling	392
8.11	Summary	404
9	Sensitivity Analysis and Shape Optimization	425
9.1	Introduction	425
9.2	Shape Optimization Methods	426
9.3	Design Sensitivity Analysis	427
9.3.1	Derivative Potential Formulation	427
9.3.2	Derivative Displacement Formulation	429
9.4	Numerical Examples	431
9.4.1	Sensitivities of a Square Plate with an Elliptical Hole	431

9.4.2	Sensitivities of Elastic Ring	432
9.4.3	A Plate Subjected to Uniaxial Tension	432
9.5	Structural Optimization	435
9.5.1	Optimum Shape of a Tank with Internal Pressure	435
9.6	Shape Identification	435
9.6.1	Identification of an Elliptical Cavity	437
9.6.2	Acoustic Scattering in Fluid-Solid	439
9.7	Flaw Identification	443
9.7.1	Potential Formulation	445
9.7.2	Elasticity Formulation	446
9.8	Examples	448
9.8.1	Curved Crack Identification	448
9.8.2	Identification of an Elliptical Crack	448
9.9	Bone Remodelling	449
9.9.1	Medullary Pin	452
9.10	Summary	453
10	Assembled Structures	461
10.1	Introduction	461
10.2	Structures Reinforced with Beams	462
10.2.1	Two-dimensional Formulations	462
10.2.2	Plate Bending Formulations	466
10.3	Patch Attachments	470
10.3.1	Two-dimensional Riveted Joints Model	471
10.3.2	Plate Bending Formulations	472
10.3.3	Adhesively Bonded Patches	475
10.4	Examples	478
10.4.1	Three-point Bending Concrete Beam	478
10.4.2	Single or Double Circular Patch Bonded to a Sheet	478
10.4.3	Simply Supported Plate with a Stiffener	481
10.4.4	Reinforced Cylindrical Shell	483
10.4.5	Fastened Single Patch Repair	483
10.4.6	Cylindrical Shell Panels with a Patch.	485
10.5	Summary	487
11	Numerical Integration	491
11.1	Integration of Regular Integrals	491
11.2	Near Singular Integration	493
11.2.1	Transformation of Variable Technique	493
11.2.2	Singularity Subtraction Technique	497
11.3	Weakly Singular Integration	497
11.3.1	Two-dimensional Problems	497
11.3.2	Three-dimensional Problems	502
11.4	Strongly Singular	508
11.4.1	Basic Definitions	508
11.4.2	Two-dimensional Problems	510
11.4.3	Three-dimensional Problems	513
11.5	Hypersingular Integrals	517
11.5.1	Basic Definitions	517

11.5.2 Two-dimensional Problems	518
11.5.3 Three-dimensional Problems	519
11.6 Summary	520
A Complex Stress Functions	525
A.1 Airy Stress Functions	525
A.2 Muskhelishvili Stress Functions	525
A.3 Complex Stress Function Method	527
B Limiting Process for Boundary Stress Equation	531
B.1 Enriched Element Approach	535
C Limit Integrals For Shallow Shells	539
C.1 The Displacement Integral Equations	539
C.2 The Bending Stress Resultant Integral Equations	541
C.3 The Shear Stress Resultant Integral Equation	543
D Particular Solutions For Shear Deformable Plates	547
D.1 Plate Bending	547
D.2 Two-dimensional Plane Stress	549
E Fundamental Solutions for Elastodynamics	553
E.1 Fundamental Solutions for Time Domain	553
E.2 Convolved Fundamental Solutions	556
E.2.1 Two-dimensional Problems	556
E.2.2 Three-dimensional Problems	558
E.3 Fundamental Solutions for Laplace Transform Domain	562
E.3.1 Behaviour of Transformed Fundamental Solutions	564
F Shape Functions of Brick Cell Elements	569
F.1 Continuous cells	569
F.2 Face-discontinuous cells	571
G Fast Solver For Contact Problems	573
G.1 LU decomposition	573
G.1.1 High Speed Solver for Contact Problems	573
Index	577