

Wavelets in Electromagnetics and Device Modeling

GEORGE W. PAN

Wiley Series in Microwave and Optical Engineering
Kai Chang, Series Editor

Contents

Preface	xv
1 Notations and Mathematical Preliminaries	1
1.1 Notations and Abbreviations	1
1.2 Mathematical Preliminaries	2
1.2.1 Functions and Integration	2
1.2.2 The Fourier Transform	4
1.2.3 Regularity	4
1.2.4 Linear Spaces	7
1.2.5 Functional Spaces	8
1.2.6 Sobolev Spaces	10
1.2.7 Bases in Hilbert Space H	11
1.2.8 Linear Operators	12
Bibliography	14
2 Intuitive Introduction to Wavelets	15
2.1 Technical History and Background	15
2.1.1 Historical Development	15
2.1.2 When Do Wavelets Work?	16
2.1.3 A Wave Is a Wave but What Is a Wavelet?	17
2.2 What Can Wavelets Do in Electromagnetics and Device Modeling?	18
2.2.1 Potential Benefits of Using Wavelets	18
2.2.2 Limitations and Future Direction of Wavelets	19
2.3 The Haar Wavelets and Multiresolution Analysis	20
	vii

2.4 How Do Wavelets Work?	23
Bibliography	28
3 Basic Orthogonal Wavelet Theory	30
3.1 Multiresolution Analysis	30
3.2 Construction of Scalets $\varphi(\tau)$	32
3.2.1 Franklin Scalet	32
3.2.2 Battle-Lemarie Scalets	39
3.2.3 Preliminary Properties of Scalets	40
3.3 Wavelet $\psi(\tau)$	42
3.4 Franklin Wavelet	48
3.5 Properties of Scalets $\hat{\varphi}(\omega)$	51
3.6 Daubechies Wavelets	56
3.7 Coifman Wavelets (Coiflets)	64
3.8 Constructing Wavelets by Recursion and Iteration	69
3.8.1 Construction of Scalets	69
3.8.2 Construction of Wavelets	74
3.9 Meyer Wavelets	75
3.9.1 Basic Properties of Meyer Wavelets	75
3.9.2 Meyer Wavelet Family	83
3.9.3 Other Examples of Meyer Wavelets	92
3.10 Mallat's Decomposition and Reconstruction	92
3.10.1 Reconstruction	92
3.10.2 Decomposition	93
3.11 Problems	95
3.11.1 Exercise 1	95
3.11.2 Exercise 2	95
3.11.3 Exercise 3	97
3.11.4 Exercise 4	97
Bibliography	98
4 Wavelets in Boundary Integral Equations	100
4.1 Wavelets in Electromagnetics	100
4.2 Linear Operators	102

4.3 Method of Moments (MoM)	103
4.4 Functional Expansion of a Given Function	107
4.5 Operator Expansion: Nonstandard Form	110
4.5.1 Operator Expansion in Haar Wavelets	111
4.5.2 Operator Expansion in General Wavelet Systems	113
4.5.3 Numerical Example	114
4.6 Periodic Wavelets	120
4.6.1 Construction of Periodic Wavelets	120
4.6.2 Properties of Periodic Wavelets	123
4.6.3 Expansion of a Function in Periodic Wavelets	127
4.7 Application of Periodic Wavelets: 2D Scattering	128
4.8 Fast Wavelet Transform (FWT)	133
4.8.1 Discretization of Operation Equations	133
4.8.2 Fast Algorithm	134
4.8.3 Matrix Sparsification Using FWT	135
4.9 Applications of the FWT	140
4.9.1 Formulation	140
4.9.2 Circuit Parameters	141
4.9.3 Integral Equations and Wavelet Expansion	143
4.9.4 Numerical Results	144
4.10 Intervallic Coifman Wavelets	144
4.10.1 Intervallic Scalets	145
4.10.2 Intervallic Wavelets on $[0, 1]$	154
4.11 Lifting Scheme and Lazy Wavelets	156
4.11.1 Lazy Wavelets	156
4.11.2 Lifting Scheme Algorithm	157
4.11.3 Cascade Algorithm	159
4.12 Green's Scalets and Sampling Series	159
4.12.1 Ordinary Differential Equations (ODEs)	160
4.12.2 Partial Differential Equations (PDEs)	166
4.13 Appendix: Derivation of Intervallic Wavelets on $[0, 1]$	172
4.14 Problems	185
4.14.1 Exercise 5	185
4.14.2 Exercise 6	185
4.14.3 Exercise 7	185

4.14.4 Exercise 8	186
4.14.5 Project 1	187
Bibliography	187
5 Sampling Biorthogonal Time Domain Method (SBTD)	189
5.1 Basis FDTD Formulation	189
5.2 Stability Analysis for the FDTD	194
5.3 FDTD as Maxwell's Equations with Haar Expansion	198
5.4 FDTD with Battle-Lemarie Wavelets	201
5.5 Positive Sampling and Biorthogonal Testing Functions	205
5.6 Sampling Biorthogonal Time Domain Method	215
5.6.1 SBTD versus MRTD	215
5.6.2 Formulation	215
5.7 Stability Conditions for Wavelet-Based Methods	219
5.7.1 Dispersion Relation and Stability Analysis	219
5.7.2 Stability Analysis for the SBTD	222
5.8 Convergence Analysis and Numerical Dispersion	223
5.8.1 Numerical Dispersion	223
5.8.2 Convergence Analysis	225
5.9 Numerical Examples	228
5.10 Appendix: Operator Form of the MRTD	233
5.11 Problems	236
5.11.1 Exercise 9	236
5.11.2 Exercise 10	237
5.11.3 Project 2	237
Bibliography	238
6 Canonical Multiwavelets	240
6.1 Vector-Matrix Dilation Equation	240
6.2 Time Domain Approach	242
6.3 Construction of Multiscalelets	245
6.4 Orthogonal Multiwavelets $\check{\psi}(t)$	255
6.5 Intervallic Multiwavelets $\psi(t)$	258

6.6 Multiwavelet Expansion	261
6.7 Intervallic Dual Multiwavelets $\tilde{\psi}(t)$	264
6.8 Working Examples	269
6.9 Multiscale-Based 1D Finite Element Method (FEM)	276
6.10 Multiscale-Based Edge Element Method	280
6.11 Spurious Modes	285
6.12 Appendix	287
6.13 Problems	296
6.13.1 Exercise 11	296
Bibliography	297
7 Wavelets in Scattering and Radiation	299
7.1 Scattering from a 2D Groove	299
7.1.1 Method of Moments (MoM) Formulation	300
7.1.2 Coiflet-Based MoM	304
7.1.3 Bi-CGSTAB Algorithm	305
7.1.4 Numerical Results	305
7.2 2D and 3D Scattering Using Intervallic Coiflets	309
7.2.1 Intervallic Scalets on $[0, 1]$	309
7.2.2 Expansion in Coifman Intervallic Wavelets	312
7.2.3 Numerical Integration and Error Estimate	313
7.2.4 Fast Construction of Impedance Matrix	317
7.2.5 Conducting Cylinders, TM Case	319
7.2.6 Conducting Cylinders with Thin Magnetic Coating	322
7.2.7 Perfect Electrically Conducting (PEC) Spheroids	324
7.3 Scattering and Radiation of Curved Thin Wires	329
7.3.1 Integral Equation for Curved Thin-Wire Scatterers and Antennae	330
7.3.2 Numerical Examples	331
7.4 Smooth Local Cosine (SLC) Method	340
7.4.1 Construction of Smooth Local Cosine Basis	341
7.4.2 Formulation of 2D Scattering Problems	344
7.4.3 SLC-Based Galerkin Procedure and Numerical Results	347
7.4.4 Application of the SLC to Thin-Wire Scatterers and Antennas	355

7.5	Microstrip Antenna Arrays	357
7.5.1	Impedance Matched Source	358
7.5.2	Far-Zone Fields and Antenna Patterns	360
	Bibliography	363
8	Wavelets in Rough Surface Scattering	366
8.1	Scattering of EM Waves from Randomly Rough Surfaces	366
8.2	Generation of Random Surfaces	368
8.2.1	Autocorrelation Method	370
8.2.2	Spectral Domain Method	373
8.3	2D Rough Surface Scattering	376
8.3.1	Moment Method Formulation of 2D Scattering	376
8.3.2	Wavelet-Based Galerkin Method for 2D Scattering	380
8.3.3	Numerical Results of 2D Scattering	381
8.4	3D Rough Surface Scattering	387
8.4.1	Tapered Wave of Incidence	388
8.4.2	Formulation of 3D Rough Surface Scattering Using Wavelets	391
8.4.3	Numerical Results of 3D Scattering	394
	Bibliography	399
9	Wavelets in Packaging, Interconnects, and EMC	401
9.1	Quasi-static Spatial Formulation	402
9.1.1	What Is Quasi-static?	402
9.1.2	Formulation	403
9.1.3	Orthogonal Wavelets in $L^2([0, 1])$	406
9.1.4	Boundary Element Method and Wavelet Expansion	408
9.1.5	Numerical Examples	412
9.2	Spatial Domain Layered Green's Functions	415
9.2.1	Formulation	417
9.2.2	Prony's Method	423
9.2.3	Implementation of the Coifman Wavelets	424
9.2.4	Numerical Examples	426
9.3	Skin-Effect Resistance and Total Inductance	429
9.3.1	Formulation	431
9.3.2	Moment Method Solution of Coupled Integral Equations	433

9.3.3 Circuit Parameter Extraction	435
9.3.4 Wavelet Implementation	437
9.3.5 Measurement and Simulation Results	438
9.4 Spectral Domain Green's Function-Based Full-Wave Analysis	440
9.4.1 Basic Formulation	440
9.4.2 Wavelet Expansion and Matrix Equation	444
9.4.3 Evaluation of Sommerfeld-Type Integrals	447
9.4.4 Numerical Results and Sparsity of Impedance Matrix	451
9.4.5 Further Improvements	455
9.5 Full-Wave Edge Element Method for 3D Lossy Structures	455
9.5.1 Formulation of Asymmetric Functionals with Truncation Conditions	456
9.5.2 Edge Element Procedure	460
9.5.3 Excess Capacitance and Inductance	464
9.5.4 Numerical Examples	466
Bibliography	469
10 Wavelets in Nonlinear Semiconductor Devices	474
10.1 Physical Models and Computational Efforts	474
10.2 An Interpolating Subdivision Scheme	476
10.3 The Sparse Point Representation (SPR)	478
10.4 Interpolation Wavelets in the FDM	479
10.4.1 1D Example of the SPR Application	480
10.4.2 2D Example of the SPR Application	481
10.5 The Drift-Diffusion Model	484
10.5.1 Scaling	486
10.5.2 Discretization	487
10.5.3 Transient Solution	489
10.5.4 Grid Adaptation and Interpolating Wavelets	490
10.5.5 Numerical Results	492
10.6 Multiwavelet Based Drift-Diffusion Model	498
10.6.1 Precision and Stability versus Reynolds	499
10.6.2 MWFEM-Based 1D Simulation	502
10.7 The Boltzmann Transport Equation (BTE) Model	504
10.7.1 Why BTE?	505
10.7.2 Spherical Harmonic Expansion of the BTE	505

xiv **CONTENTS**

10.7.3	Arbitrary Order Expansion and Galerkin's Procedure	509
10.7.4	The Coupled Boltzmann-Poisson System	515
10.7.5	Numerical Results	517
	Bibliography	524
	Index	527