

# **Visual Quantum Mechanics**

**Selected Topics with  
Computer-Generated  
Animations of  
Quantum-Mechanical  
Phenomena**

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**CD-ROM  
INCLUDED**

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