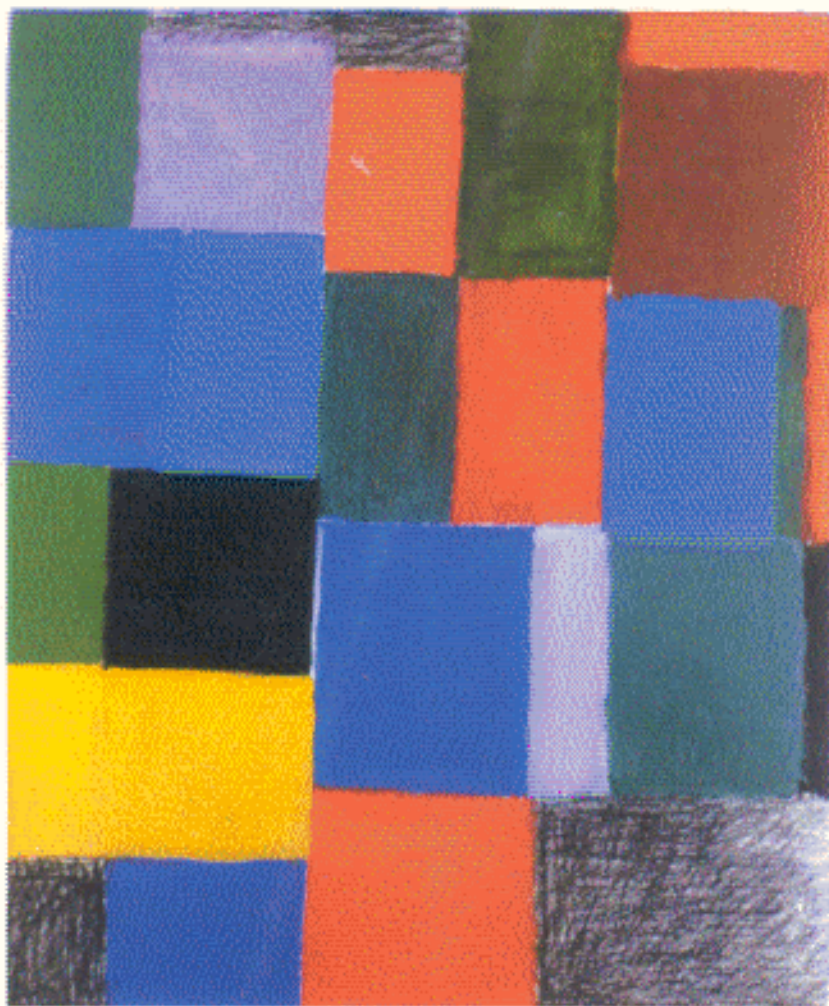


INTERNATIONAL EDITION

ELEMENTARY



LINEAR ALGEBRA

8TH  
EDITION

BERNARD KOLMAN | DAVID R. HILL

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