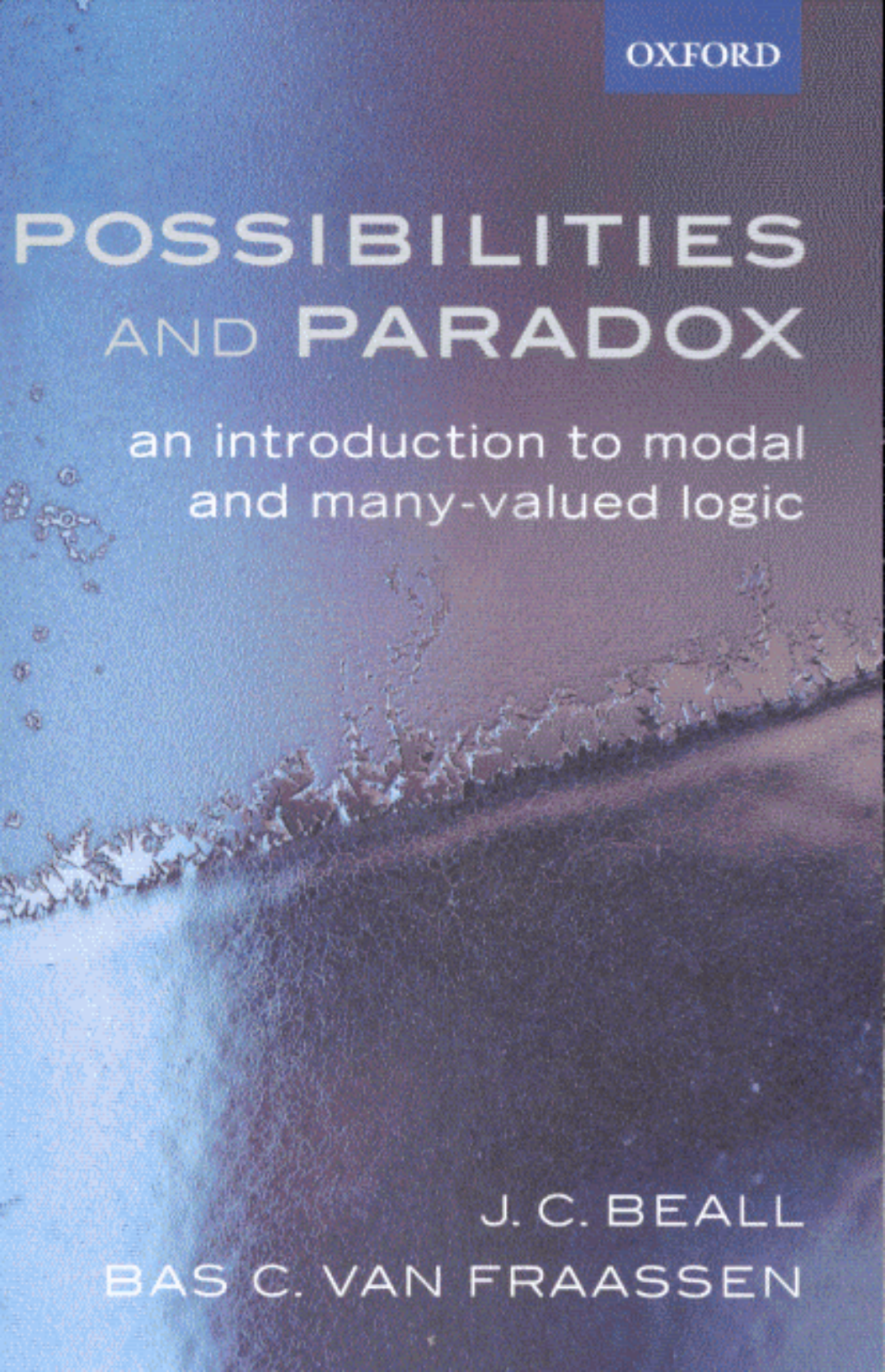




OXFORD

POSSIBILITIES AND PARADOX

an introduction to modal
and many-valued logic

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