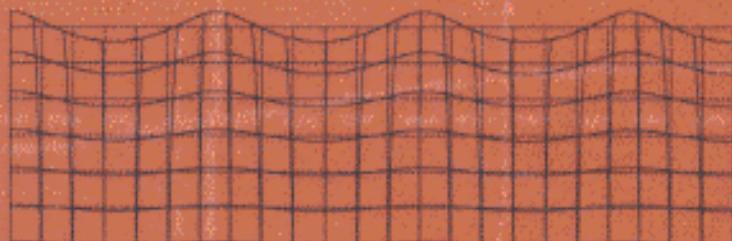


Andrew N. Cleland

# Foundations of Nanomechanics

From Solid-State Theory  
to Device Applications



Springer

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