APPLICATIONS OF MATHEMATICS

STOCHASTIC MODELLING AND APPLIED PROBABILITY

48

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Stochastic Portfolio Theory

Springer

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