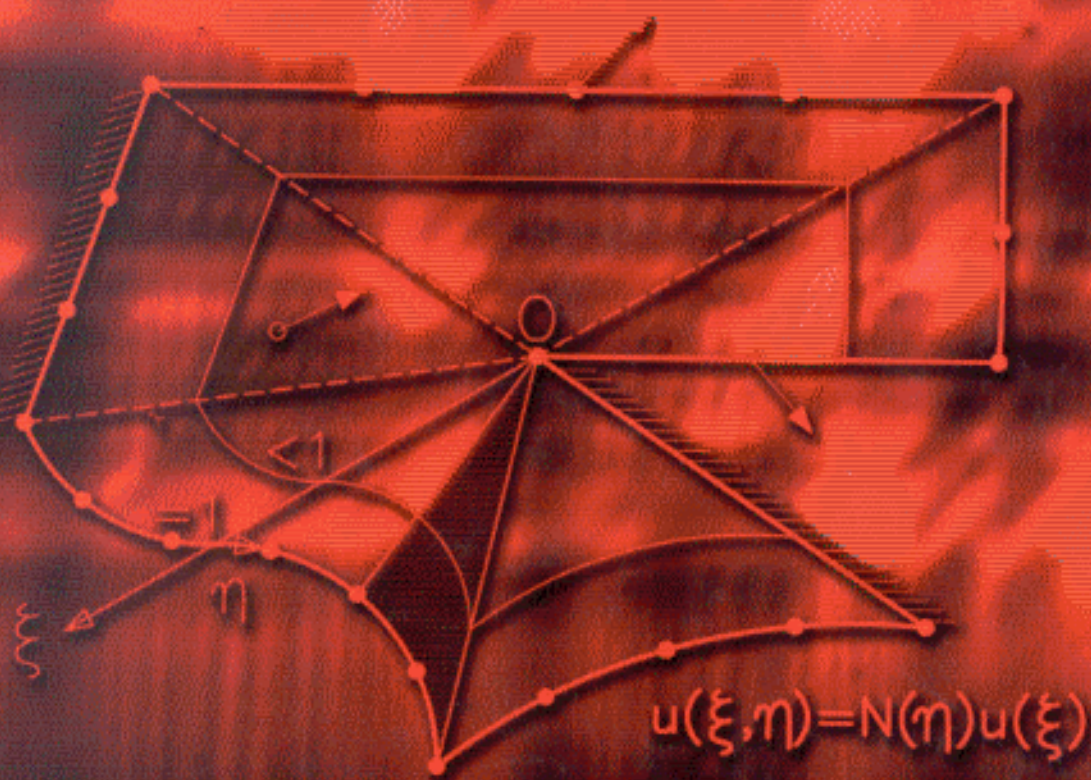


The Scaled Boundary Finite Element Method



John P. Wolf

 WILEY

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