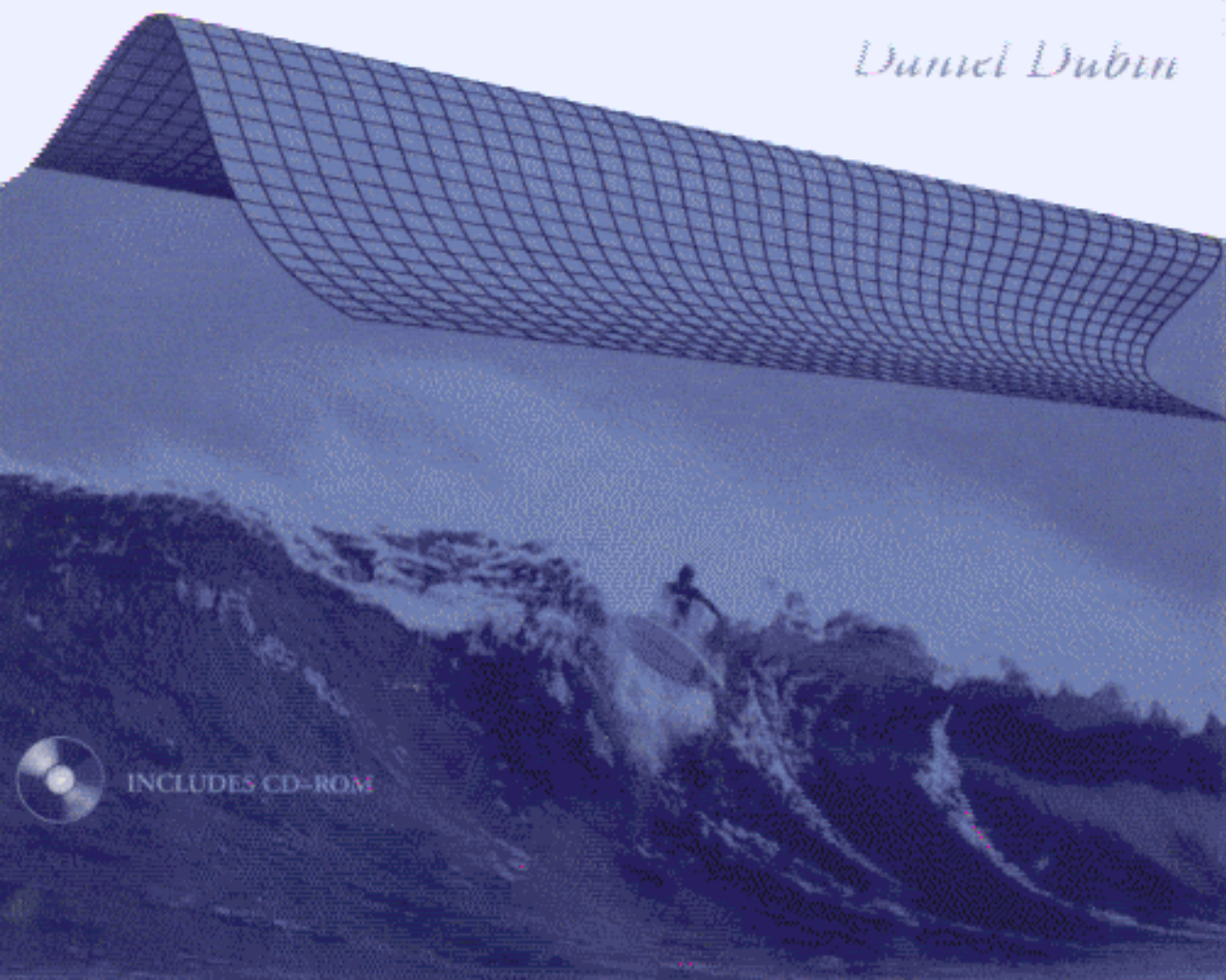


# Numerical and Analytical Methods for Scientists and Engineers Using *Mathematica*<sup>®</sup>

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