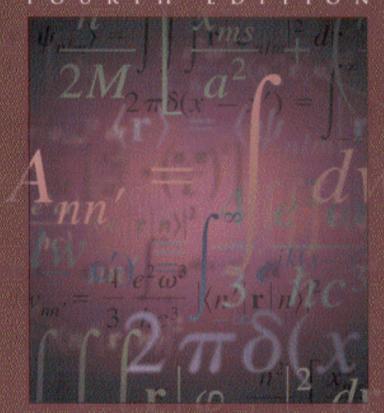
## QUANTUM MECHANICS



Richard L. Liboff

## **Contents**

|        | Pref   | ace  | vii  |
|--------|--------|--|------|
|        | List   | of Tables  | χv   |
|        | Торі   | ical Problems  | xvii |
| PART I |        | MENTARY PRINCIPLES AND APPLICATIONS TO DBLEMS IN ONE DIMENSION         | 1    |
| 1 ■    | Revi   | iew of Concepts of Classical Mechanics                                 | 3    |
|        | 1.1    | Generalized or "Good" Coordinates 3                                    |      |
|        | 1.2    | Energy, the Hamiltonian, and Angular Momentum 6                        |      |
|        | 1.3    | The State of a System 19   |      |
|        | 1.4    | Properties of the One-Dimensional Potential Function 24                |      |
| 2 ■    | l Hist | torical Review: Experiments and Theories                               | 30   |
|        | 2.1    | Dates 30   |      |
|        | 2.2    | The Work of Planck. Blackbody Radiation 31                             |      |
|        | 2.3    | The Work of Einstein. The Photoelectric Effect 36                      |      |
|        | 2.4    | The Work of Bohr. A Quantum Theory of Atomic States 39                 |      |
|        | 2.5    | Waves versus Particles 43  |      |
|        | 2.6    | The de Broglie Hypothesis and the Davisson–Germer Experiment 46        |      |
|        | 2.7    | The Work of Heisenberg. Uncertainty as a Cornerstone of Natural Law 53 |      |
|        | 2.8    | The Work of Born. Probability Waves 55                                 |      |
|        | 2.9    | Semiphilosophical Epilogue to Chapter 2 57                             | _    |

| 3 ■        |       | Postulates of Quantum Mechanics. Operators, enfunctions, and Eigenvalues | 68  |
|------------|-------|--|-----|
|            | 3.1   | Observables and Operators 68   | 00  |
|            | 3.2   | Measurement in Quantum Mechanics 74                                      |     |
|            | 3.3   | The State Function and Expectation Values 76                             |     |
|            | 3.4   | Time Development of the State Function 80                                |     |
|            | 3.5   | Solution to the Initial-Value Problem in Quantum Mechanics 84            |     |
| 4 ■        | ■ Pre | paratory Concepts. Function Spaces and                                   |     |
|            |       | mitian Operators   | 90  |
|            | 4.1   | Particle in a Box and Further Remarks on Normalization 90                |     |
|            | 4.2   | The Bohr Correspondence Principle 94                                     |     |
|            | 4.3   | Dirac Notation 97  |     |
|            | 4.4   | Hilbert Space 98   |     |
|            | 4.5   | Hermitian Operators 104  |     |
|            | 4.6   | Properties of Hermitian Operators 108                                    |     |
| 5 <b>I</b> | ∎ Sup | erposition and Compatible Observables                                    | 115 |
|            | 5.1   | The Superposition Principle 115  |     |
|            | 5.2   | Commutator Relations in Quantum Mechanics 130                            |     |
|            | 5.3   | More on the Commutator Theorem 137                                       |     |
|            | 5.4   | Commutator Relations and the Uncertainty Principle 140                   |     |
|            | 5.5   | "Complete" Sets of Commuting Observables 143                             |     |
| 6 ■        | I Tim | e Development, Conservation Theorems, and Parity                         | 152 |
|            | 6.1   | Time Development of State Functions 152                                  |     |
|            | 6.2   | Time Development of Expectation Values 168                               |     |
|            | 6.3   | Conservation of Energy, Linear and Angular Momentum 171                  |     |
|            | 6.4   | Conservation of Parity 176   |     |
| 7          | ■ Add | litional One-Dimensional Problems. Bound and                             |     |
|            | _     | oound States   | 187 |
|            | 7.1   | General Properties of the One-Dimensional Schrödinger Equation 187       |     |
|            | 7.2   | The Harmonic Oscillator 190  |     |
|            | 7.3   | Eigenfunctions of the Harmonic Oscillator Hamiltonian 198                |     |
|            | 7.4   | The Harmonic Oscillator in Momentum Space 211                            |     |
|            | 7.5   | Unbound States 216   |     |
|            | 76    | One-Dimensional Barrier Problems 222                                     |     |

ix Contents

|         | 7.7        | The Rectangular Barrier. Tunneling 228   |          |
|---------|------------|--|----------|
|         | 7.8        | The Ramsauer Effect 235  |          |
|         | 7.9        | Kinetic Properties of a Wave Packet Scattered from a Potential Barrier 241                 |          |
|         | 7.10       | The WKB Approximation 243  |          |
|         | 7.11       | Principle of Least Action and Feynman's Path Integral Formulation 268                      |          |
| 8 ■     |            | e Potential Well, Periodic Lattice, and Some Simple  | e<br>278 |
|         |            | lems with Two Degrees of Freedom   | 2/0      |
|         | 8.1        | The Finite Potential Well 278  |          |
|         | 8.2        | Periodic Lattice. Energy Gaps 289  |          |
|         | 8.3<br>8.4 | Standing Waves at the Band Edges 307   |          |
|         | 8.4        | Brief Qualitative Description of the Theory of Conduction in Solids 313                    |          |
|         | 8.5        | Two Beads on a Wire and a Particle in a Two-Dimensional Box                                | 317      |
|         | 8.6        | Two-Dimensional Harmonic Oscillator 324  |          |
|         | 8.7        | Linear Combination of Atomic Orbitals (LCAO) Approximation 331                             |          |
|         | 8.8        | Density of States in Various Dimensions 336  |          |
| PART II | AND        | THER DEVELOPMENT OF THE THEORY  O APPLICATIONS TO PROBLEMS IN  EE DIMENSIONS               | 347      |
| 9 ■     | Ang        | ular Momentum  | 349      |
|         | 9.1        | Basic Properties 349   |          |
|         | 9.2        | Eigenvalues of the Angular Momentum Operators 358  |          |
|         | 9.3        | Eigenfunctions of the Orbital Angular Momentum Operators $\hat{L}^2$ and $\hat{L}_z = 367$ |          |
|         | 9.4        | Addition of Angular Momentum 386   |          |
|         | 9.5        | Total Angular Momentum for Two or More Electrons 396                                       |          |
| 10 ■    | Prob       | olems in Three Dimensions  | 404      |
|         | 10.1       | The Free Particle in Cartesian Coordinates 404   |          |
|         | 10.2       | The Free Particle in Spherical Coordinates 410   |          |
|         | 10.3       | The Free-Particle Radial Wavefunction 415  |          |
|         | 10.4       | A Charged Particle in a Magnetic Field 430   |          |
|         | 10.5       | The Two-Particle Problem 436   |          |
|         | 10.6       | The Hydrogen Atom 446  |          |

| _   |     |     |
|-----|-----|-----|
| Cor | nto | nte |
| CU  | ILC |     |

| 10<br>10       | .7 Elementary Theory of Radiation 463 .8 Thomas–Fermi Model 472 |     |
|----------------|---|-----|
| 11 <b>■</b> El | ements of Matrix Mechanics. Spin Wavefunctions                  | 480 |
| 11             |   | .00 |
| 11             |   |     |
| 11             |   | 402 |
| 11             |   | 7/2 |
| 11             |   |     |
|                | .6 The Pauli Spin Matrices 512                                  |     |
| 11             |   |     |
| 11             |   |     |
|                | 9 Precession of an Electron in a Magnetic Field 527             |     |
|                | 10 The Addition of Two Spins 536                                |     |
|                | 11 The Density Matrix 543                                       |     |
| · ·            | 12 Other "Pictures" in Quantum Mechanics 553                    |     |
|                | 13 Polarization States, EPR Revisited 558                       |     |
|                | .14 The Transfer Matrix 571                                     |     |
|                |   |     |
| 12 <b>■</b> Aı | oplication to Atomic, Molecular, Solid-State, and               |     |
|                | uclear Physics. Elements of Quantum Statistics                  | 579 |
| 12             |   | 373 |
| 12             |   |     |
| 12             |   |     |
| 12             | 1   |     |
| 12             |   |     |
| 12             |   |     |
| 12             |   |     |
| 12             | , ,   |     |
| 12.            | Superfluidity 630   |     |
| 12             | -   |     |
|                | 10 Elements of Nuclear Physics. The Deuteron and Isospin 669    |     |
|                | 20 Maria de l'acteur l'hystos. The Deuteron and Isospin 007     |     |
| 13 <b>■</b> Pe | rturbation Theory   | 681 |
| 13.            | ,   | 001 |
| 13             | 1 , 2   |     |
| 13.            | , , , , , , , , , , , , , , , , , , ,                           |     |
| 13.            |   |     |
| 13.            |   |     |
|                | 6 Harmonic Perturbation 712                                     |     |

|   | Conte  | nts   | хi        |
|---|--------|---|-----------|
|   | Conto  |   | <b>7.</b> |
|   | 13.7   | Application of Harmonic Perturbation Theory 718                           |           |
|   | 13.8   |   |           |
|   | 13.9   |   |           |
|   | 13.10  | Hartree–Fock Model 757  |           |
| 14 ■                                      | Scatt  | tering in Three Dimensions  | 762       |
|   | 14.1   | Partial Waves 762   |           |
|   | 14.2   | S-Wave Scattering 770   |           |
|   | 14.3   | Center-of-Mass Frame 774  |           |
|   | 14.4   | The Born Approximation 777  |           |
|   | 14.5   | Atomic-Radiative Absorption Cross Section 782                             |           |
|   | 14.6   | Elements of Formal Scattering Theory. The Lippmann–Schwinger Equation 785 |           |
| 15 ■                                      | Rela   | tivistic Quantum Mechanics  | 793       |
|   | 15.1   | Preliminary Remarks 793   |           |
|   | 15.2   | Klein-Gordon Equation 798   |           |
|   | 15.3   | Dirac Equation 800  |           |
|   | 15.4   | Electron Magnetic Moment 806  |           |
|   | 15.5   | Covariant Description 810   |           |
| 16 ■                                      | Qua    | ntum Computing  | 817       |
|   | 16.1   | Binary Number System 817  |           |
|   | 16.2   | Logic Gates 823   |           |
|   | 16.3   | Turing Machine and Complexity Classes 830                                 |           |
|   | 16.4   | Qubits and Quantum Logic Gates 832  |           |
|   | List   | of Symbols  | 843       |
|   | APP    | ENDIXES   | 847       |
| A <b>=</b>                                | l Δddi | itional Remarks on the $\hat{x}$ and $\hat{p}$ Representations            | 849       |
| A <b>=</b>                                | . Auu  | monai kemaiks on the x and p kepresemations                               | UTJ       |
| B ■ Spin and Statistics                   |        |   | 853       |
| C ■ Representations of the Delta Function |        |   | 857       |
| D ■ Differential Vector Relations         |        |   | 861       |

