

PULSE WIDTH MODULATION FOR POWER CONVERTERS

PRINCIPLES AND PRACTICE

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$$\begin{aligned}
 i(t) = & \frac{I_{on}}{2} + \sum_{n=1}^{\infty} [A_{on} \cos(n[\omega_o t + \theta_o]) + B_{on} \sin(n[\omega_o t + \theta_o])] \\
 & + \sum_{m=1}^{\infty} [A_{m0} \cos(m[\omega_c t + \theta_c]) + B_{m0} \sin(m[\omega_c t + \theta_c])] \\
 & + \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[A_{mn} \cos(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]) \right. \\
 & \quad \left. + B_{mn} \sin(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]) \right]
 \end{aligned}$$

DC Offset

Fundamental Component & Baseband Harmonics

Carrier Harmonics

Sideband Harmonics

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