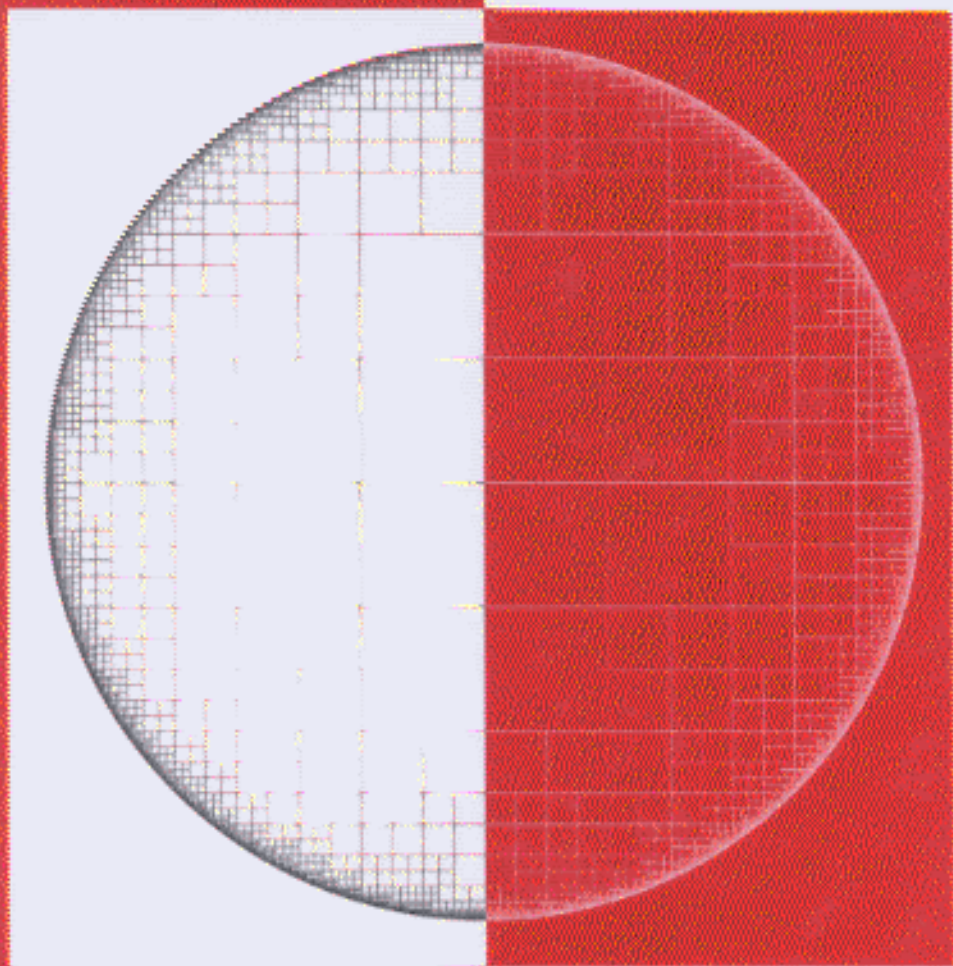


Classical and Modern Fourier Analysis



Loukas Grafakos

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