Optimization Principles

Practical Applications to the Operation and Markets of the Electric Power Industry

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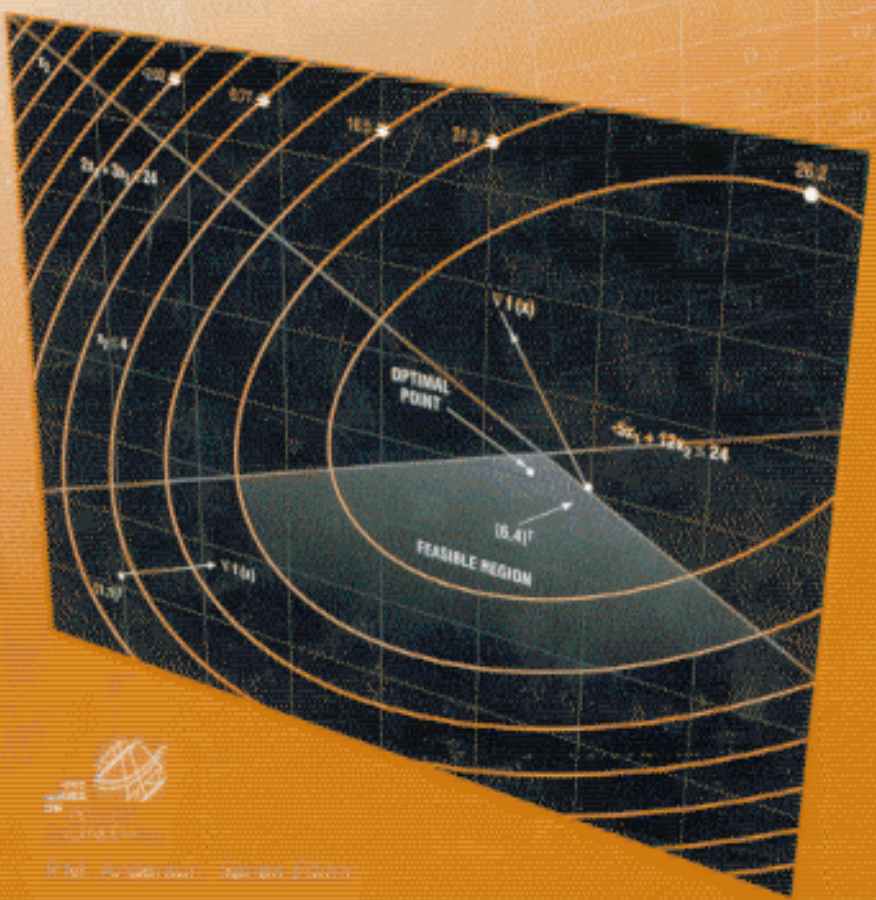

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