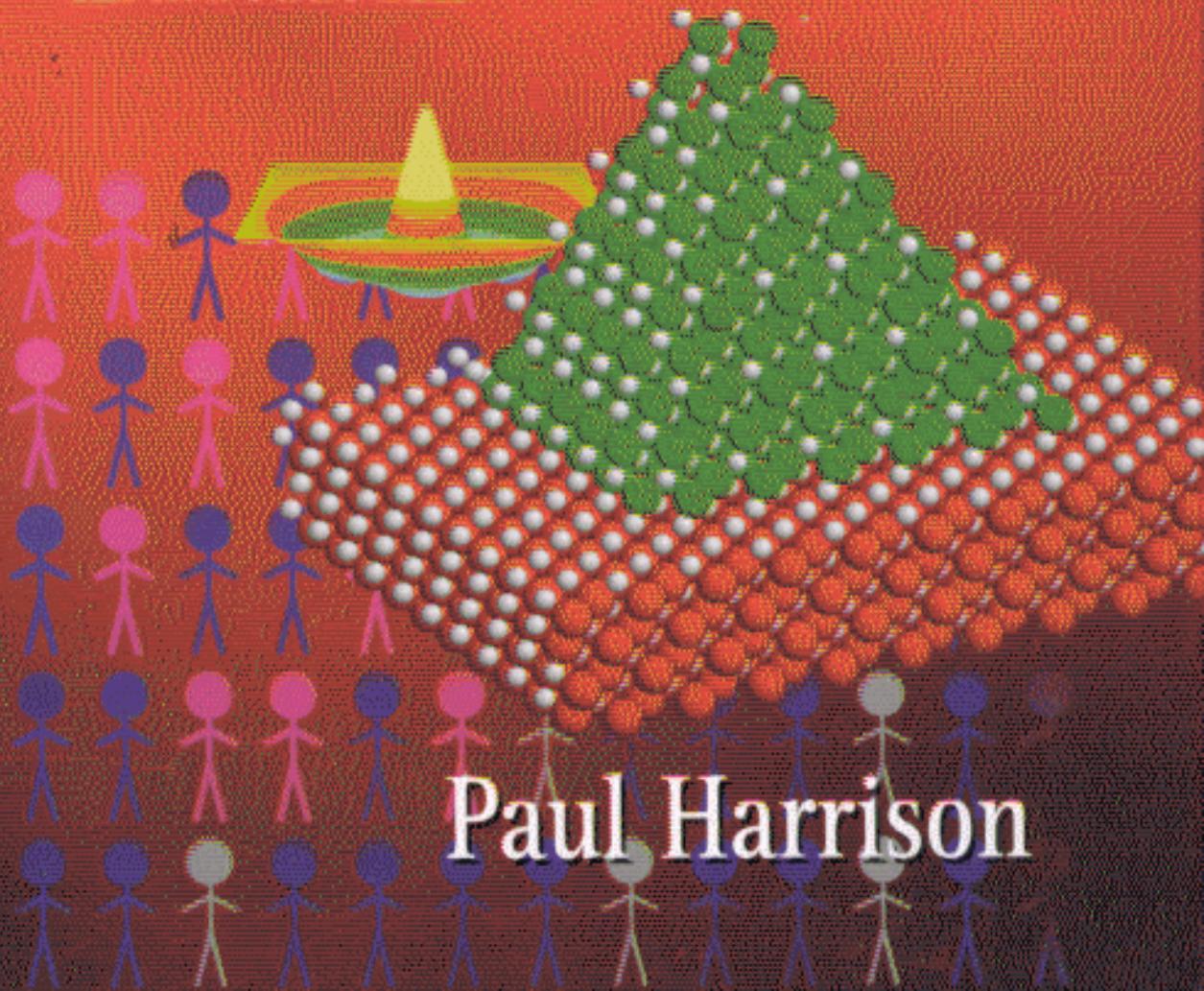


# Computational Methods in Physics, Chemistry and Biology

## An Introduction



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# *Contents*

Preface	xi
Acknowledgements	xiii
About the author	xv
About the book	xvii
Introduction	xix
1 Numerical Solutions to Schrödinger's Equation	1
1.1 Particles and Classical Mechanics	1
1.2 Waves and Quantum Mechanics	6
1.3 Analytical Solutions to Schrödinger's Equation	11
1.4 The Shooting Method	16
1.5 Initial Conditions	19
1.6 Computational Implementation	20
1.7 Application to Parabolic Potentials	22
1.8 Improved Energy Eigenvalues	25
1.9 Further Convergence Tests: Pushing Accuracy to its Limits	28
1.10 Deduction of the Wave Functions	31
1.11 Three-Dimensional Potentials	33
1.12 Application to the Hydrogen Atom	34

2	Approximate Methods	45
2.1	Small Changes	45
2.2	Non-Degenerate Perturbation Theory	46
2.3	Computing the First-Order Correction	48
2.4	Computing the Second-Order Correction	52
2.5	Two-Dimensional Perturbing Potentials	55
2.6	The Variational Method	58
2.7	Application to the Hydrogen Atom	60
2.8	Application to Biased Quantum Wells	64
3	Matrix Methods	71
3.1	Basis sets	71
3.2	Expansion	74
3.3	Formation of the Matrix Equation	76
3.4	Solution of the Matrix Equation	77
3.5	Computational Implementation for Quantum Wells	78
3.6	Convergence Tests: Application to Parabolic Potentials	82
3.7	Solution of Schrödinger's Equation in Several Dimensions	84
3.8	Degenerate Perturbation Theory	86
4	Deterministic Simulations	91
4.1	Complex Time Dependent Systems	91
4.2	Classical Mechanics and Space Rockets	92
4.3	Simulation of Launch Trajectory	96
4.4	Verification of Computational Parameters	98
4.5	Fun with Simulations	100
4.6	Introduction to Diffusion	101
4.7	Theory	104
4.8	Boundary Conditions	105
4.9	Numerical Implementation	105
4.10	Convergence Tests	107
4.11	Constant Diffusion Coefficients and Universality	108
4.12	Non-linear Diffusion	109
5	Stochastic Simulations	115
5.1	Stochastic or Monte Carlo Simulations	115
5.2	Random Numbers	116
5.3	How Random is Random?	117

5.4	Monte Carlo Simulations of Electron Scattering	118
5.5	Tests and Limits	121
5.6	Simulations of Three-Level Systems	123
5.7	Changing Probabilities and Pauli Exclusion	125
6	Percolation Theory	131
6.1	Complex Many-Body Interacting Systems	131
6.2	Disease Propagation	132
6.3	Computational Implementation	133
6.4	The Dependence of Infection upon Immunisation	137
6.5	The Limit of Large Populations and Critical Densities	140
6.6	Mixed Magnetic Phases	142
7	Evolutionary Methods	149
7.1	Genetic Algorithms	149
7.2	The Fitness Function	150
7.3	Random (Monte Carlo) Generation of Genomes	150
7.4	Reproduction and Natural Selection	156
7.5	Implementation of a Genetic Algorithm	160
7.6	Mutation and Conclusion	164
8	Molecular Dynamics	169
8.1	Modelling Molecules and Crystals	169
8.2	Atom–Atom Interactions and Interaction Potentials	170
8.3	Simulations of Atomic Ensembles	173
8.4	Computational Implementation	175
8.5	Initial Simulations and Convergence Tests	179
8.6	Larger Ensembles and Reproducing Nature	180
8.7	Non-spherical Interaction Potentials	182
8.8	Organic Molecules	183
	Appendix A: FORTRAN implementation of the shooting method	189
	Appendix B: $\nabla^2$ in spherical polar coordinates	191
	Appendix C: A comment on the computer sourcecodes	193
	Appendix D: Note for tutors	195