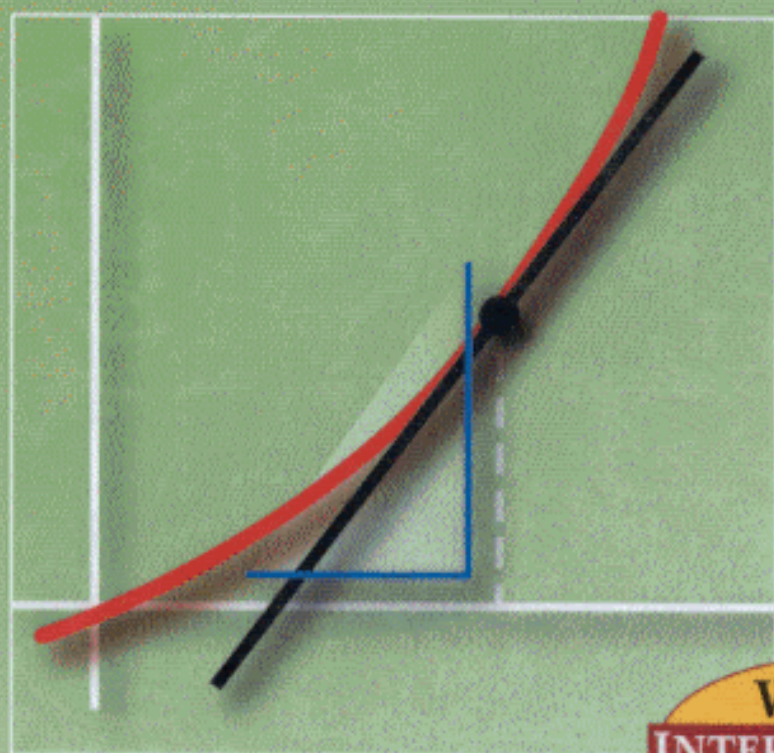


ELEMENTARY NUMERICAL ANALYSIS

THIRD EDITION



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ATKINSON • HAN

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