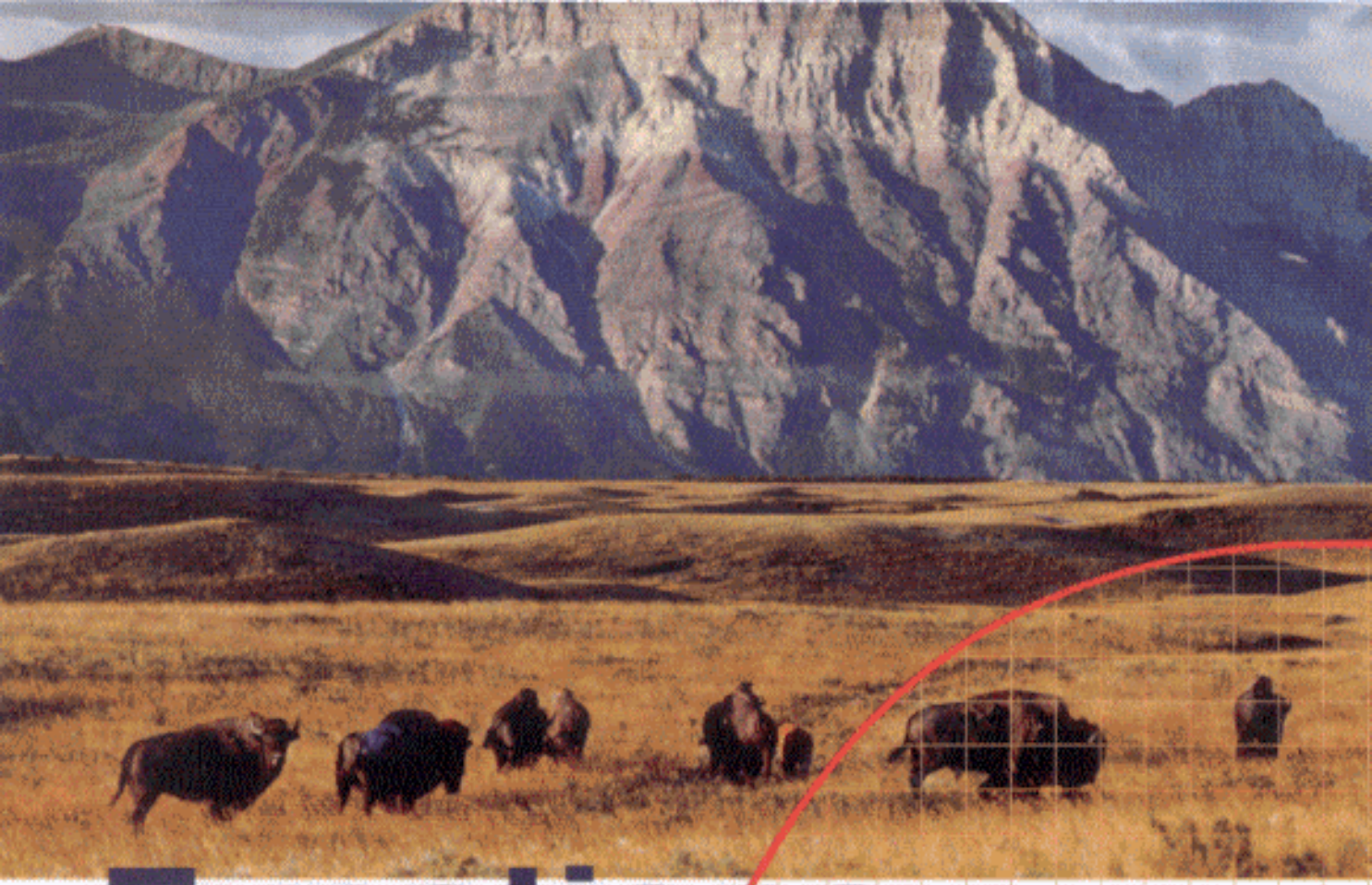

Functions and Change

A Modeling Approach to College Algebra

Second Edition

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