## A First Course in Dynanics

WITH A PANORAMA OF RECENT DEVELOPMENTS
Boris Hasselblatt and Anatole Katok



## Contents

Preface	
1 Introduction	1
1.1 Dynamics	1
1.2 Dynamics in Nature	4
1.3 Dynamics in Mathematics	19
PART 1. A COURSE IN DYNAMICS: FROM SIMPLE TO	
COMPLICATED BEHAVIOR	29
2 Systems with Stable Asymptotic Behavior	31
2.1 Linear Maps and Linearization	31
2.2 Contractions in Euclidean Space	32
2.3 Nondecreasing Maps of an Interval and Bifurcations	45
2.4 Differential Equations	49
2.5 Quadratic Maps	57
2.6 Metric Spaces	61
2.7 Fractals	69
3 Linear Maps and Linear Differential Equations	
3.1 Linear Maps in the Plane	73
3.2 Linear Differential Equations in the Plane	86
3.3 Linear Maps and Differential Equations in Higher Dimension	90
4 Recurrence and Equidistribution on the Circle	96
4.1 Rotations of the Circle	96
4.2 Some Applications of Density and Uniform Distribution	109
4.3 Invertible Circle Maps	123
4.4 Cantor Phenomena	135
5 Recurrence and Equidistribution in Higher Dimension	143
5.1 Translations and Linear Flows on the Torus	143
5.2 Applications of Translations and Linear Flows	152

6	Conse	vative Systems	
	6.1	Preservation of Phase Volume and Recurrence	
	6.2	Newtonian Systems of Classical Mechanics	
	6.3	Billiards: Definition and Examples	
		Convex Billiards	
7	Simple	Systems with Complicated Orbit Structure	
		Growth of Periodic Points	
		Topological Transitivity and Chaos	
		Coding	
		More Examples of Coding	
		Uniform Distribution	
		Independence, Entropy, Mixing	
R		y and Chaos	
•	_	Dimension of a Compact Space	
		Topological Entropy	
		Applications and Extensions	
		1. pproceeding and execusions	
PART 2. PANORAMA OF DYNAMICAL SYSTEMS			
9	Simple	Dynamics as a Tool	
	9.1	Introduction	
	9.2	Implicit- and Inverse-Function Theorems in Euclidean Space	
		Persistence of Transverse Fixed Points	
	9.4	Solutions of Differential Equations	
		Hyperbolicity	
10	Hypert	polic Dynamics	
		Hyperbolic Sets	
	10.2	Orbit Structure and Orbit Growth	
	10.3	Coding and Mixing	
		Statistical Properties	
		Nonuniformly Hyperbolic Dynamical Systems	
11		atic Maps	
	11.1	Preliminaries	
	11.2	Development of Simple Behavior Beyond the First Bifurcation	
		Onset of Complexity	
		Hyperbolic and Stochastic Behavior	
12	Homod	linic Tangles	
		Nonlinear Horseshoes	
	12.2	Homoclinic Points	
	12.3	The Appearance of Horseshoes	
		The Importance of Horseshoes	
		Detecting Homoclinic Tangles: The Poincaré-Melnikov	
		Method	
	12.6	Homoclinic Tangencies	

13	Strange Attractors	331	
	13.1 Familiar Attractors	331	
	13.2 The Solenoid	333	
	13.3 The Lorenz Attractor	335	
14	Variational Methods, Twist Maps, and Closed Geodesics	342	
	14.1 The Variational Method and Birkhoff Periodic Orbits		
	for Billiards	342	
	14.2 Birkhoff Periodic Orbits and Aubry-Mather Theory		
	for Twist Maps	346	
	14.3 Invariant Circles and Regions of Instability	357	
	14.4 Periodic Points for Maps of the Cylinder	360	
	14.5 Geodesics on the Sphere	362	
15	Dynamics, Number Theory, and Diophantine Approximation	365	
	15.1 Uniform Distribution of the Fractional Parts of Polynomials	365	
	15.2 Continued Fractions and Rational Approximation	369	
	15.3 The Gauß Map	374	
	15.4 Homogeneous Dynamics, Geometry, and Number Theory	377	
	15.5 Quadratic Forms in Three Variables	383	
Reading			
APPENDIX			
	A.1 Metric Spaces	389	
	A.2 Differentiability	400	
	A.3 Riemann Integration in Metric Spaces	401	
Hin	Hints and Answers		
Sol	Solutions		
Index			
mu	<b>CA</b>	419	