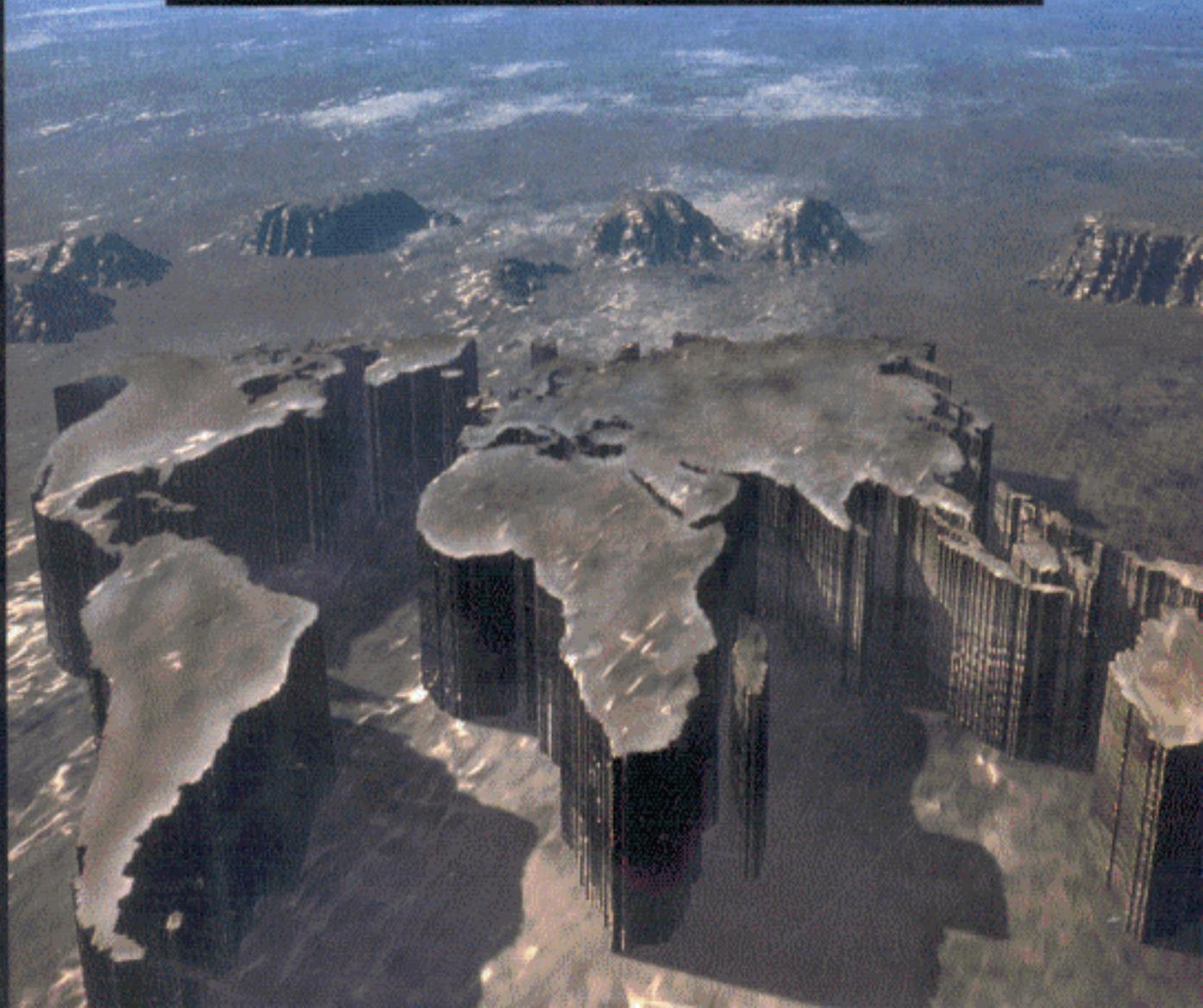


A D Y N A M I C A P P R O A C H

# ELEMENTARY MATHEMATICAL MODELING

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