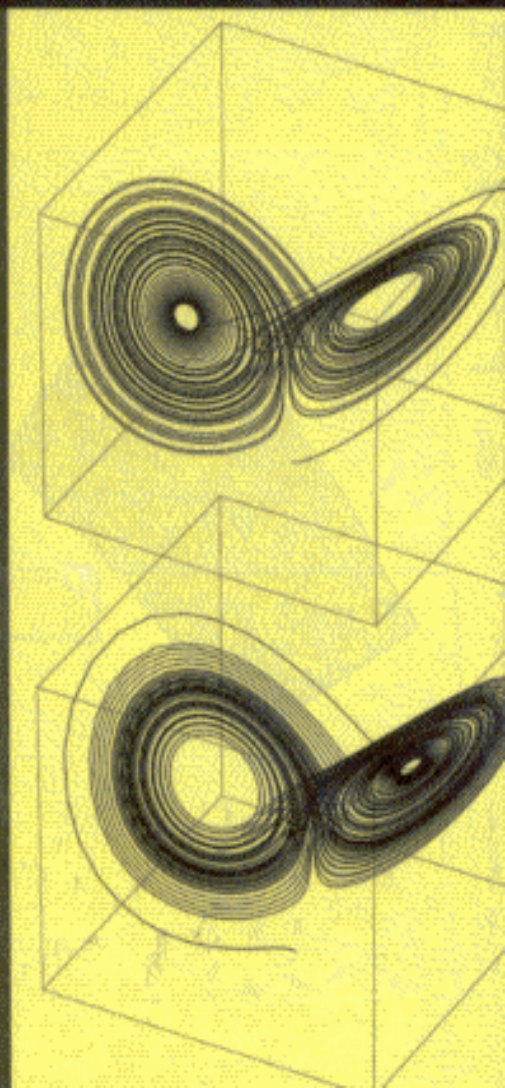




Differential Equations, Dynamical Systems & An Introduction to Chaos

Second Edition

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