



Donald T. Greenwood

# Advanced Dynamics



CAMBRIDGE

# Contents

## *Preface*

page ix

---

## **1 Introduction to particle dynamics**

---

|     |                                     |    |
|-----|-------------------------------------|----|
| 1.1 | Particle motion                     | 1  |
| 1.2 | Systems of particles                | 15 |
| 1.3 | Constraints and configuration space | 34 |
| 1.4 | Work, energy and momentum           | 40 |
| 1.5 | Impulse response                    | 53 |
| 1.6 | Bibliography                        | 65 |
| 1.7 | Problems                            | 65 |

---

## **2 Lagrange's and Hamilton's equations**

---

|     |   |     |
|-----|---|-----|
| 2.1 | D'Alembert's principle and Lagrange's equations | 73  |
| 2.2 | Hamilton's equations                            | 84  |
| 2.3 | Integrals of the motion                         | 91  |
| 2.4 | Dissipative and gyroscopic forces               | 99  |
| 2.5 | Configuration space and phase space             | 110 |
| 2.6 | Impulse response, analytical methods            | 117 |
| 2.7 | Bibliography                                    | 130 |
| 2.8 | Problems  | 130 |

---

## **3 Kinematics and dynamics of a rigid body**

---

|     |                           |     |
|-----|---------------------------|-----|
| 3.1 | Kinematical preliminaries | 140 |
| 3.2 | Dyadic notation           | 159 |

---

|     |                           |     |
|-----|---------------------------|-----|
| 3.3 | Basic rigid body dynamics | 162 |
| 3.4 | Impulsive motion          | 188 |
| 3.5 | Bibliography              | 205 |
| 3.6 | Problems                  | 206 |

---

**4 Equations of motion: differential approach**

|      |  |     |
|------|--|-----|
| 4.1  | Quasi-coordinates and quasi-velocities | 217 |
| 4.2  | Maggi's equation                       | 219 |
| 4.3  | The Boltzmann–Hamel equation           | 226 |
| 4.4  | The general dynamical equation         | 234 |
| 4.5  | A fundamental equation                 | 246 |
| 4.6  | The Gibbs–Appell equation              | 254 |
| 4.7  | Constraints and energy rates           | 261 |
| 4.8  | Summary of differential methods        | 274 |
| 4.9  | Bibliography                           | 278 |
| 4.10 | Problems                               | 279 |

**5 Equations of motion: integral approach**

|     |  |     |
|-----|--|-----|
| 5.1 | Hamilton's principle                               | 289 |
| 5.2 | Transpositional relations                          | 296 |
| 5.3 | The Boltzmann–Hamel equation, transpositional form | 304 |
| 5.4 | The central equation                               | 307 |
| 5.5 | Suslov's principle                                 | 315 |
| 5.6 | Summary of integral methods                        | 322 |
| 5.7 | Bibliography                                       | 323 |
| 5.8 | Problems   | 324 |

**6 Introduction to numerical methods**

|     |                             |     |
|-----|-----------------------------|-----|
| 6.1 | Interpolation               | 329 |
| 6.2 | Numerical integration       | 335 |
| 6.3 | Numerical stability         | 349 |
| 6.4 | Frequency response methods  | 356 |
| 6.5 | Kinematic constraints       | 364 |
| 6.6 | Energy and momentum methods | 383 |

**Contents**

---

|                  |     |
|------------------|-----|
| 6.7 Bibliography | 396 |
| 6.8 Problems     | 396 |

**Appendix**

---

|                         |     |
|-------------------------|-----|
| A.1 Answers to problems | 401 |
| <i>Index</i>            | 421 |