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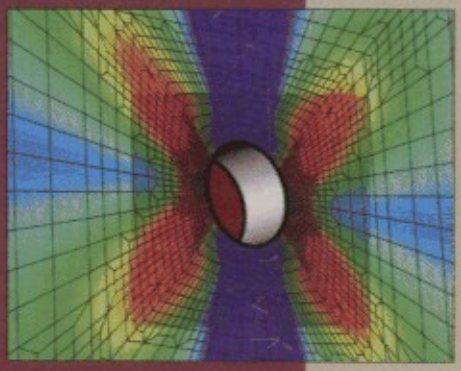

Encyclopedia of Computational Mechanics

Volume 1 Fundamentals
Volume 2 Solids and Structures
Volume 3 Fluids

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