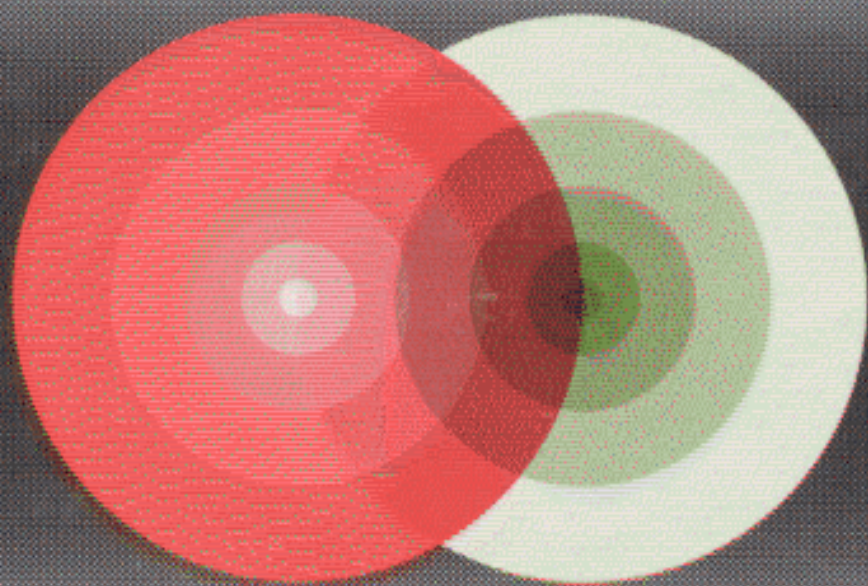


Geometry, Topology and Quantum Field Theory

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Fundamental Theories of Physics

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