

# Computational Rheology for Pipeline and Annular Flow

*Non-Newtonian Flow Modeling  
for Drilling and Production,  
and Flow Assurance Methods  
in Subsea Pipeline Design*

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