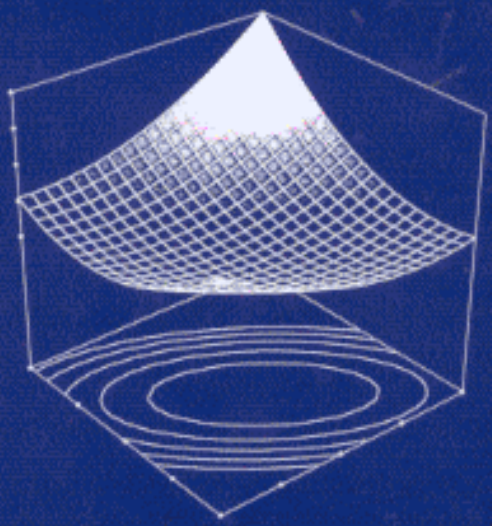
WILEY

Response Surface Methodology

Process and Product Optimization Using Designed Experiments

SECOND EDITION



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WILEY SERIES IN PROBABILITY AND STATISTICS

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