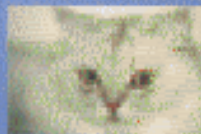


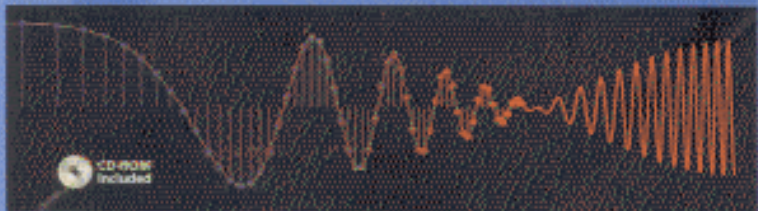
**International Edition**

# **Signal Processing First**

**James H. McClellan  
Ronald W. Schafer  
Mark A. Yoder**



CD-ROM  
Included



# Contents

## PREFACE

xiv

## 1 Introduction 1

|     |                                                  |   |
|-----|--------------------------------------------------|---|
| 1-1 | Mathematical Representation of Signals . . . . . | 2 |
| 1-2 | Mathematical Representation of Systems . . . . . | 4 |
| 1-3 | Thinking About Systems . . . . .                 | 5 |
| 1-4 | The Next Step . . . . .                          | 6 |

## 2 Sinusoids 7

|     |                                               |    |
|-----|-----------------------------------------------|----|
| 2-1 | Tuning Fork Experiment . . . . .              | 8  |
| 2-2 | Review of Sine and Cosine Functions . . . . . | 9  |
| 2-3 | Sinusoidal Signals . . . . .                  | 11 |

|       |                                               |    |
|-------|-----------------------------------------------|----|
| 2-3.1 | Relation of Frequency to Period               | 12 |
| 2-3.2 | Phase Shift and Time Shift                    | 13 |
| 2-4   | Sampling and Plotting Sinusoids               | 15 |
| 2-5   | Complex Exponentials and Phasors              | 17 |
| 2-5.1 | Review of Complex Numbers                     | 17 |
| 2-5.2 | Complex Exponential Signals                   | 18 |
| 2-5.3 | The Rotating Phasor Interpretation            | 19 |
| 2-5.4 | Inverse Euler Formulas                        | 21 |
| 2-6   | Phasor Addition                               | 22 |
| 2-6.1 | Addition of Complex Numbers                   | 23 |
| 2-6.2 | Phasor Addition Rule                          | 23 |
| 2-6.3 | Phasor Addition Rule: Example                 | 24 |
| 2-6.4 | MATLAB Demo of Phasors                        | 25 |
| 2-6.5 | Summary of the Phasor Addition Rule           | 26 |
| 2-7   | Physics of the Tuning Fork                    | 27 |
| 2-7.1 | Equations from Laws of Physics                | 27 |
| 2-7.2 | General Solution to the Differential Equation | 29 |
| 2-7.3 | Listening to Tones                            | 29 |
| 2-8   | Time Signals: More Than Formulas              | 29 |
| 2-9   | Summary and Links                             | 30 |
| 2-10  | Problems                                      | 31 |

## 3 Spectrum Representation 36

|       |                                    |    |
|-------|------------------------------------|----|
| 3-1   | The Spectrum of a Sum of Sinusoids | 36 |
| 3-1.1 | Notation Change                    | 38 |
| 3-1.2 | Graphical Plot of the Spectrum     | 38 |
| 3-2   | Beat Notes                         | 39 |
| 3-2.1 | Multiplication of Sinusoids        | 39 |
| 3-2.2 | Beat Note Waveform                 | 40 |
| 3-2.3 | Amplitude Modulation               | 41 |
| 3-3   | Periodic Waveforms                 | 43 |
| 3-3.1 | Synthetic Vowel                    | 44 |
| 3-3.2 | Example of a Nonperiodic Signal    | 45 |
| 3-4   | Fourier Series                     | 47 |
| 3-4.1 | Fourier Series: Analysis           | 48 |
| 3-4.2 | Fourier Series Derivation          | 48 |
| 3-5   | Spectrum of the Fourier Series     | 50 |

|         |                                                    |    |
|---------|----------------------------------------------------|----|
| 3-6     | Fourier Analysis of Periodic Signals . . . . .     | 51 |
| 3-6.1   | The Square Wave . . . . .                          | 52 |
| 3-6.1.1 | DC Value of a Square Wave . . . . .                | 53 |
| 3-6.2   | Spectrum for a Square Wave . . . . .               | 53 |
| 3-6.3   | Synthesis of a Square Wave . . . . .               | 54 |
| 3-6.4   | Triangle Wave . . . . .                            | 55 |
| 3-6.5   | Synthesis of a Triangle Wave . . . . .             | 56 |
| 3-6.6   | Convergence of Fourier Synthesis . . . . .         | 57 |
| 3-7     | Time-Frequency Spectrum . . . . .                  | 57 |
| 3-7.1   | Stepped Frequency . . . . .                        | 59 |
| 3-7.2   | Spectrogram Analysis . . . . .                     | 59 |
| 3-8     | Frequency Modulation: Chirp Signals . . . . .      | 60 |
| 3-8.1   | Chirp or Linearly Swept Frequency . . . . .        | 60 |
| 3-8.2   | A Closer Look at Instantaneous Frequency . . . . . | 62 |
| 3-9     | Summary and Links . . . . .                        | 63 |
| 3-10    | Problems . . . . .                                 | 64 |

## 4 Sampling and Aliasing

71

|       |                                                                   |    |
|-------|-------------------------------------------------------------------|----|
| 4-1   | Sampling . . . . .                                                | 71 |
| 4-1.1 | Sampling Sinusoidal Signals . . . . .                             | 73 |
| 4-1.2 | The Concept of Aliasing . . . . .                                 | 75 |
| 4-1.3 | Spectrum of a Discrete-Time Signal . . . . .                      | 76 |
| 4-1.4 | The Sampling Theorem . . . . .                                    | 77 |
| 4-1.5 | Ideal Reconstruction . . . . .                                    | 78 |
| 4-2   | Spectrum View of Sampling and Reconstruction . . . . .            | 79 |
| 4-2.1 | Spectrum of a Discrete-Time Signal Obtained by Sampling . . . . . | 79 |
| 4-2.2 | Over-Sampling . . . . .                                           | 79 |
| 4-2.3 | Aliasing Due to Under-Sampling . . . . .                          | 81 |
| 4-2.4 | Folding Due to Under-Sampling . . . . .                           | 82 |
| 4-2.5 | Maximum Reconstructed Frequency . . . . .                         | 83 |
| 4-3   | Strobe Demonstration . . . . .                                    | 84 |
| 4-3.1 | Spectrum Interpretation . . . . .                                 | 87 |
| 4-4   | Discrete-to-Continuous Conversion . . . . .                       | 88 |
| 4-4.1 | Interpolation with Pulses . . . . .                               | 88 |
| 4-4.2 | Zero-Order Hold Interpolation . . . . .                           | 89 |
| 4-4.3 | Linear Interpolation . . . . .                                    | 90 |

|       |                                            |    |
|-------|--------------------------------------------|----|
| 4-4.4 | Cubic Spline Interpolation . . . . .       | 90 |
| 4-4.5 | Over-Sampling Aids Interpolation . . . . . | 91 |
| 4-4.6 | Ideal Bandlimited Interpolation . . . . .  | 92 |
| 4-5   | The Sampling Theorem . . . . .             | 93 |
| 4-6   | Summary and Links . . . . .                | 94 |
| 4-7   | Problems . . . . .                         | 96 |

## 5 FIR Filters

101

|         |                                                 |     |
|---------|-------------------------------------------------|-----|
| 5-1     | Discrete-Time Systems . . . . .                 | 102 |
| 5-2     | The Running-Average Filter . . . . .            | 102 |
| 5-3     | The General FIR Filter . . . . .                | 105 |
| 5-3.1   | An Illustration of FIR Filtering . . . . .      | 106 |
| 5-3.2   | The Unit Impulse Response . . . . .             | 107 |
| 5-3.2.1 | Unit Impulse Sequence . . . . .                 | 107 |
| 5-3.2.2 | Unit Impulse Response Sequence . . . . .        | 108 |
| 5-3.2.3 | The Unit-Delay System . . . . .                 | 109 |
| 5-3.3   | Convolution and FIR Filters . . . . .           | 110 |
| 5-3.3.1 | Computing the Output of a Convolution . . . . . | 110 |
| 5-3.3.2 | Convolution in MATLAB . . . . .                 | 111 |
| 5-4     | Implementation of FIR Filters . . . . .         | 111 |
| 5-4.1   | Building Blocks . . . . .                       | 111 |
| 5-4.1.1 | Multiplier . . . . .                            | 112 |
| 5-4.1.2 | Adder . . . . .                                 | 112 |
| 5-4.1.3 | Unit Delay . . . . .                            | 112 |
| 5-4.2   | Block Diagrams . . . . .                        | 113 |
| 5-4.2.1 | Other Block Diagrams . . . . .                  | 113 |
| 5-4.2.2 | Internal Hardware Details . . . . .             | 115 |
| 5-5     | Linear Time-Invariant (LTI) Systems . . . . .   | 115 |
| 5-5.1   | Time Invariance . . . . .                       | 116 |
| 5-5.2   | Linearity . . . . .                             | 117 |
| 5-5.3   | The FIR Case . . . . .                          | 117 |
| 5-6     | Convolution and LTI Systems . . . . .           | 118 |
| 5-6.1   | Derivation of the Convolution Sum . . . . .     | 118 |
| 5-6.2   | Some Properties of LTI Systems . . . . .        | 120 |
| 5-6.2.1 | Convolution as an Operator . . . . .            | 121 |
| 5-6.2.2 | Commutative Property of Convolution . . . . .   | 121 |
| 5-6.2.3 | Associative Property of Convolution . . . . .   | 121 |

|      |                                    |     |
|------|------------------------------------|-----|
| 5-7  | Cascaded LTI Systems . . . . .     | 122 |
| 5-8  | Example of FIR Filtering . . . . . | 124 |
| 5-9  | Summary and Links . . . . .        | 126 |
| 5-10 | Problems . . . . .                 | 126 |

## 6 Frequency Response of FIR Filters 130

|       |                                                                |     |
|-------|----------------------------------------------------------------|-----|
| 6-1   | Sinusoidal Response of FIR Systems . . . . .                   | 130 |
| 6-2   | Superposition and the Frequency Response . . . . .             | 132 |
| 6-3   | Steady-State and Transient Response . . . . .                  | 135 |
| 6-4   | Properties of the Frequency Response . . . . .                 | 137 |
| 6-4.1 | Relation to Impulse Response and Difference Equation . . . . . | 137 |
| 6-4.2 | Periodicity of $H(e^{j\omega})$ . . . . .                      | 138 |
| 6-4.3 | Conjugate Symmetry . . . . .                                   | 138 |
| 6-5   | Graphical Representation of the Frequency Response . . . . .   | 139 |
| 6-5.1 | Delay System . . . . .                                         | 139 |
| 6-5.2 | First-Difference System . . . . .                              | 140 |
| 6-5.3 | A Simple Lowpass Filter . . . . .                              | 142 |
| 6-6   | Cascaded LTI Systems . . . . .                                 | 143 |
| 6-7   | Running-Average Filtering . . . . .                            | 145 |
| 6-7.1 | Plotting the Frequency Response . . . . .                      | 146 |
| 6-7.2 | Cascade of Magnitude and Phase . . . . .                       | 148 |
| 6-7.3 | Experiment: Smoothing an Image . . . . .                       | 149 |
| 6-8   | Filtering Sampled Continuous-Time Signals . . . . .            | 151 |
| 6-8.1 | Example: Lowpass Averager . . . . .                            | 152 |
| 6-8.2 | Interpretation of Delay . . . . .                              | 154 |
| 6-9   | Summary and Links . . . . .                                    | 155 |
| 6-10  | Problems . . . . .                                             | 157 |

## 7 z-Transforms 163

|       |                                              |     |
|-------|----------------------------------------------|-----|
| 7-1   | Definition of the z-Transform . . . . .      | 164 |
| 7-2   | The z-Transform and Linear Systems . . . . . | 165 |
| 7-2.1 | The z-Transform of an FIR Filter . . . . .   | 166 |

|       |                                                                               |     |
|-------|-------------------------------------------------------------------------------|-----|
| 7-3   | Properties of the $z$ -Transform . . . . .                                    | 167 |
| 7-3.1 | The Superposition Property of the $z$ -Transform . . . . .                    | 168 |
| 7-3.2 | The Time-Delay Property of the $z$ -Transform . . . . .                       | 168 |
| 7-3.3 | A General $z$ -Transform Formula . . . . .                                    | 169 |
| 7-4   | The $z$ -Transform as an Operator . . . . .                                   | 169 |
| 7-4.1 | Unit-Delay Operator . . . . .                                                 | 169 |
| 7-4.2 | Operator Notation . . . . .                                                   | 170 |
| 7-4.3 | Operator Notation in Block Diagrams . . . . .                                 | 170 |
| 7-5   | Convolution and the $z$ -Transform . . . . .                                  | 171 |
| 7-5.1 | Cascading Systems . . . . .                                                   | 173 |
| 7-5.2 | Factoring $z$ -Polynomials . . . . .                                          | 174 |
| 7-5.3 | Deconvolution . . . . .                                                       | 175 |
| 7-6   | Relationship Between the $z$ -Domain and the $\hat{\omega}$ -Domain . . . . . | 175 |
| 7-6.1 | The $z$ -Plane and the Unit Circle . . . . .                                  | 176 |
| 7-6.2 | The Zeros and Poles of $H(z)$ . . . . .                                       | 177 |
| 7-6.3 | Significance of the Zeros of $H(z)$ . . . . .                                 | 178 |
| 7-6.4 | Nulling Filters . . . . .                                                     | 179 |
| 7-6.5 | Graphical Relation Between $z$ and $\hat{\omega}$ . . . . .                   | 180 |
| 7-7   | Useful Filters . . . . .                                                      | 181 |
| 7-7.1 | The $L$ -Point Running-Sum Filter . . . . .                                   | 181 |
| 7-7.2 | A Complex Bandpass Filter . . . . .                                           | 183 |
| 7-7.3 | A Bandpass Filter with Real Coefficients . . . . .                            | 185 |
| 7-8   | Practical Bandpass Filter Design . . . . .                                    | 186 |
| 7-9   | Properties of Linear-Phase Filters . . . . .                                  | 189 |
| 7-9.1 | The Linear-Phase Condition . . . . .                                          | 189 |
| 7-9.2 | Locations of the Zeros of FIR Linear-Phase Systems . . . . .                  | 189 |
| 7-10  | Summary and Links . . . . .                                                   | 190 |
| 7-11  | Problems . . . . .                                                            | 191 |

## 8 IIR Filters

196

|       |                                                           |     |
|-------|-----------------------------------------------------------|-----|
| 8-1   | The General IIR Difference Equation . . . . .             | 197 |
| 8-2   | Time-Domain Response . . . . .                            | 198 |
| 8-2.1 | Linearity and Time Invariance of IIR Filters . . . . .    | 199 |
| 8-2.2 | Impulse Response of a First-Order IIR System . . . . .    | 200 |
| 8-2.3 | Response to Finite-Length Inputs . . . . .                | 201 |
| 8-2.4 | Step Response of a First-Order Recursive System . . . . . | 202 |
| 8-3   | System Function of an IIR Filter . . . . .                | 204 |
| 8-3.1 | The General First-Order Case . . . . .                    | 205 |

|         |                                                      |     |
|---------|------------------------------------------------------|-----|
| 8-3.2   | The System Function and Block-Diagram Structures     | 206 |
| 8-3.2.1 | Direct Form I Structure                              | 206 |
| 8-3.2.2 | Direct Form II Structure                             | 207 |
| 8-3.2.3 | The Transposed Form Structure                        | 208 |
| 8-3.3   | Relation to the Impulse Response                     | 209 |
| 8-3.4   | Summary of the Method                                | 209 |
| 8-4     | Poles and Zeros                                      | 210 |
| 8-4.1   | Poles or Zeros at the Origin or Infinity             | 211 |
| 8-4.2   | Pole Locations and Stability                         | 211 |
| 8-5     | Frequency Response of an IIR Filter                  | 212 |
| 8-5.1   | Frequency Response using MATLAB                      | 213 |
| 8-5.2   | Three-Dimensional Plot of a System Function          | 214 |
| 8-6     | Three Domains                                        | 216 |
| 8-7     | The Inverse $z$ -Transform and Some Applications     | 216 |
| 8-7.1   | Revisiting the Step Response of a First-Order System | 217 |
| 8-7.2   | A General Procedure for Inverse $z$ -Transformation  | 218 |
| 8-8     | Steady-State Response and Stability                  | 220 |
| 8-9     | Second-Order Filters                                 | 223 |
| 8-9.1   | $z$ -Transform of Second-Order Filters               | 223 |
| 8-9.2   | Structures for Second-Order IIR Systems              | 224 |
| 8-9.3   | Poles and Zeros                                      | 225 |
| 8-9.4   | Impulse Response of a Second-Order IIR System        | 226 |
| 8-9.4.1 | Real Poles                                           | 227 |
| 8-9.5   | Complex Poles                                        | 228 |
| 8-10    | Frequency Response of Second-Order IIR Filter        | 231 |
| 8-10.1  | Frequency Response via MATLAB                        | 232 |
| 8-10.2  | 3-dB Bandwidth                                       | 232 |
| 8-10.3  | Three-Dimensional Plot of System Functions           | 233 |
| 8-11    | Example of an IIR Lowpass Filter                     | 236 |
| 8-12    | Summary and Links                                    | 237 |
| 8-13    | Problems                                             | 238 |

## 9 Continuous-Time Signals and LTI Systems

245

|       |                                   |     |
|-------|-----------------------------------|-----|
| 9-1   | Continuous-Time Signals           | 246 |
| 9-1.1 | Two-Sided Infinite-Length Signals | 246 |
| 9-1.2 | One-Sided Signals                 | 247 |
| 9-1.3 | Finite-Length Signals             | 248 |

|       |                                                            |     |
|-------|------------------------------------------------------------|-----|
| 9-2   | The Unit Impulse . . . . .                                 | 248 |
| 9-2.1 | Sampling Property of the Impulse . . . . .                 | 250 |
| 9-2.2 | Mathematical Rigor . . . . .                               | 252 |
| 9-2.3 | Engineering Reality . . . . .                              | 252 |
| 9-2.4 | Derivative of the Unit Step . . . . .                      | 252 |
| 9-3   | Continuous-Time Systems . . . . .                          | 254 |
| 9-3.1 | Some Basic Continuous-Time Systems . . . . .               | 254 |
| 9-3.2 | Continuous-Time Outputs . . . . .                          | 255 |
| 9-3.3 | Analogous Discrete-Time Systems . . . . .                  | 255 |
| 9-4   | Linear Time-Invariant Systems . . . . .                    | 255 |
| 9-4.1 | Time-Invariance . . . . .                                  | 256 |
| 9-4.2 | Linearity . . . . .                                        | 256 |
| 9-4.3 | The Convolution Integral . . . . .                         | 257 |
| 9-4.4 | Properties of Convolution . . . . .                        | 259 |
| 9-5   | Impulse Responses of Basic LTI Systems . . . . .           | 260 |
| 9-5.1 | Integrator . . . . .                                       | 260 |
| 9-5.2 | Differentiator . . . . .                                   | 261 |
| 9-5.3 | Ideal Delay . . . . .                                      | 261 |
| 9-6   | Convolution of Impulses . . . . .                          | 261 |
| 9-7   | Evaluating Convolution Integrals . . . . .                 | 263 |
| 9-7.1 | Delayed Unit-Step Input . . . . .                          | 263 |
| 9-7.2 | Evaluation of Discrete Convolution . . . . .               | 267 |
| 9-7.3 | Square-Pulse Input . . . . .                               | 268 |
| 9-7.4 | Very Narrow Square Pulse Input . . . . .                   | 269 |
| 9-7.5 | Discussion of Convolution Examples . . . . .               | 270 |
| 9-8   | Properties of LTI Systems . . . . .                        | 270 |
| 9-8.1 | Cascade and Parallel Combinations . . . . .                | 270 |
| 9-8.2 | Differentiation and Integration of Convolution . . . . .   | 272 |
| 9-8.3 | Stability and Causality . . . . .                          | 273 |
| 9-9   | Using Convolution to Remove Multipath Distortion . . . . . | 276 |
| 9-10  | Summary . . . . .                                          | 278 |
| 9-11  | Problems . . . . .                                         | 279 |

## 10 Frequency Response

285

|          |                                                           |     |
|----------|-----------------------------------------------------------|-----|
| 10-1     | The Frequency Response Function for LTI Systems . . . . . | 285 |
| 10-1.1   | Plotting the Frequency Response . . . . .                 | 287 |
| 10-1.1.1 | Logarithmic Plot . . . . .                                | 288 |
| 10-1.2   | Magnitude and Phase Changes . . . . .                     | 288 |

|        |                                        |     |
|--------|----------------------------------------|-----|
| 10-2   | Response to Real Sinusoidal Signals    | 289 |
| 10-2.1 | Cosine Inputs                          | 290 |
| 10-2.2 | Symmetry of $H(j\omega)$               | 290 |
| 10-2.3 | Response to a General Sum of Sinusoids | 293 |
| 10-2.4 | Periodic Input Signals                 | 294 |
| 10-3   | Ideal Filters                          | 295 |
| 10-3.1 | Ideal Delay System                     | 295 |
| 10-3.2 | Ideal Lowpass Filter                   | 296 |
| 10-3.3 | Ideal Highpass Filter                  | 297 |
| 10-3.4 | Ideal Bandpass Filter                  | 297 |
| 10-4   | Application of Ideal Filters           | 298 |
| 10-5   | Time-Domain or Frequency-Domain?       | 300 |
| 10-6   | Summary/Future                         | 301 |
| 10-7   | Problems                               | 302 |

## 11 Continuous-Time Fourier Transform 307

|          |                                                    |     |
|----------|----------------------------------------------------|-----|
| 11-1     | Definition of the Fourier Transform                | 308 |
| 11-2     | Fourier Transform and the Spectrum                 | 310 |
| 11-2.1   | Limit of the Fourier Series                        | 310 |
| 11-3     | Existence and Convergence of the Fourier Transform | 312 |
| 11-4     | Examples of Fourier Transform Pairs                | 313 |
| 11-4.1   | Right-Sided Real Exponential Signals               | 313 |
| 11-4.1.1 | Bandwidth and Decay Rate                           | 314 |
| 11-4.2   | Rectangular Pulse Signals                          | 314 |
| 11-4.3   | Bandlimited Signals                                | 316 |
| 11-4.4   | Impulse in Time or Frequency                       | 317 |
| 11-4.5   | Sinusoids                                          | 318 |
| 11-4.6   | Periodic Signals                                   | 319 |
| 11-5     | Properties of Fourier Transform Pairs              | 322 |
| 11-5.1   | The Scaling Property                               | 322 |
| 11-5.2   | Symmetry Properties of Fourier Transform Pairs     | 324 |
| 11-6     | The Convolution Property                           | 326 |
| 11-6.1   | Frequency Response                                 | 326 |
| 11-6.2   | Fourier Transform of a Convolution                 | 327 |
| 11-6.3   | Examples of the Use of the Convolution Property    | 328 |
| 11-6.3.1 | Convolution of Two Bandlimited Functions           | 328 |
| 11-6.3.2 | Product of Two Sinc Functions                      | 329 |
| 11-6.3.3 | Partial Fraction Expansions                        | 330 |

|        |                                                              |     |
|--------|--------------------------------------------------------------|-----|
| 11-7   | Basic LTI Systems . . . . .                                  | 332 |
| 11-7.1 | Time Delay . . . . .                                         | 332 |
| 11-7.2 | Differentiation . . . . .                                    | 333 |
| 11-7.3 | Systems Described by Differential Equations . . . . .        | 334 |
| 11-8   | The Multiplication Property . . . . .                        | 335 |
| 11-8.1 | The General Signal Multiplication Property . . . . .         | 335 |
| 11-8.2 | The Frequency Shifting Property . . . . .                    | 336 |
| 11-9   | Table of Fourier Transform Properties and Pairs . . . . .    | 337 |
| 11-10  | Using the Fourier Transform for Multipath Analysis . . . . . | 337 |
| 11-11  | Summary . . . . .                                            | 341 |
| 11-12  | Problems . . . . .                                           | 342 |

## 12 Filtering, Modulation, and Sampling 346

|          |                                                              |     |
|----------|--------------------------------------------------------------|-----|
| 12-1     | Linear Time-Invariant Systems . . . . .                      | 346 |
| 12-1.1   | Cascade and Parallel Configurations . . . . .                | 347 |
| 12-1.2   | Ideal Delay . . . . .                                        | 348 |
| 12-1.3   | Frequency Selective Filters . . . . .                        | 351 |
| 12-1.3.1 | Ideal Lowpass Filter . . . . .                               | 351 |
| 12-1.3.2 | Other Ideal Frequency Selective Filters . . . . .            | 352 |
| 12-1.4   | Example of Filtering in the Frequency-Domain . . . . .       | 353 |
| 12-1.5   | Compensation for the Effect of an LTI Filter . . . . .       | 355 |
| 12-2     | Sinewave Amplitude Modulation . . . . .                      | 358 |
| 12-2.1   | Double-Sideband Amplitude Modulation . . . . .               | 358 |
| 12-2.2   | DSBAM with Transmitted Carrier (DSBAM-TC) . . . . .          | 362 |
| 12-2.3   | Frequency Division Multiplexing . . . . .                    | 366 |
| 12-3     | Sampling and Reconstruction . . . . .                        | 368 |
| 12-3.1   | The Sampling Theorem and Aliasing . . . . .                  | 368 |
| 12-3.2   | Bandlimited Signal Reconstruction . . . . .                  | 370 |
| 12-3.3   | Bandlimited Interpolation . . . . .                          | 372 |
| 12-3.4   | Ideal C-to-D and D-to-C Converters . . . . .                 | 373 |
| 12-3.5   | The Discrete-Time Fourier Transform . . . . .                | 375 |
| 12-3.6   | The Inverse DTFT . . . . .                                   | 376 |
| 12-3.7   | Discrete-Time Filtering of Continuous-Time Signals . . . . . | 377 |
| 12-4     | Summary . . . . .                                            | 380 |
| 12-5     | Problems . . . . .                                           | 381 |

**13 Computing the Spectrum****389**

|       |                                                           |     |
|-------|-----------------------------------------------------------|-----|
| 13-1  | Finite Fourier Sum . . . . .                              | 390 |
| 13-2  | Too Many Fourier Transforms? . . . . .                    | 391 |
|       | 13-2.1 Relation of the DTFT to the CTFT . . . . .         | 392 |
|       | 13-2.2 Relation of the DFT to the DTFT . . . . .          | 393 |
|       | 13-2.3 Relation of the DFT to the CTFT . . . . .          | 393 |
| 13-3  | Time-Windowing . . . . .                                  | 393 |
| 13-4  | Analysis of a Sum of Sinusoids . . . . .                  | 395 |
|       | 13-4.1 DTFT of a Windowed Sinusoid . . . . .              | 398 |
| 13-5  | Discrete Fourier Transform . . . . .                      | 399 |
|       | 13-5.1 The Inverse DFT . . . . .                          | 400 |
|       | 13-5.2 Summary of the DFT Representation . . . . .        | 401 |
|       | 13-5.3 The Fast Fourier Transform (FFT) . . . . .         | 402 |
|       | 13-5.4 Negative Frequencies and the DFT . . . . .         | 402 |
|       | 13-5.5 DFT Example . . . . .                              | 403 |
| 13-6  | Spectrum Analysis of Finite-Length Signals . . . . .      | 405 |
| 13-7  | Spectrum Analysis of Periodic Signals . . . . .           | 407 |
| 13-8  | The Spectrogram . . . . .                                 | 408 |
|       | 13-8.1 Spectrogram Display . . . . .                      | 409 |
|       | 13-8.2 Spectrograms in MATLAB . . . . .                   | 410 |
|       | 13-8.3 Spectrogram of a Sampled Periodic Signal . . . . . | 410 |
|       | 13-8.4 Resolution of the Spectrogram . . . . .            | 411 |
|       | 13-8.4.1 Resolution Experiment . . . . .                  | 412 |
|       | 13-8.5 Spectrogram of a Musical Scale . . . . .           | 413 |
|       | 13-8.6 Spectrogram of a Speech Signal . . . . .           | 415 |
|       | 13-8.7 Filtered Speech . . . . .                          | 418 |
| 13-9  | The Fast Fourier Transform (FFT) . . . . .                | 420 |
|       | 13-9.1 Derivation of the FFT . . . . .                    | 420 |
|       | 13-9.1.1 FFT Operation Count . . . . .                    | 421 |
| 13-10 | Summary and Links . . . . .                               | 423 |
| 13-11 | Problems . . . . .                                        | 424 |

**A Complex Numbers****427**

|     |                                        |     |
|-----|----------------------------------------|-----|
| A-1 | Introduction . . . . .                 | 428 |
| A-2 | Notation for Complex Numbers . . . . . | 428 |
|     | A-2.1 Rectangular Form . . . . .       | 428 |

|     |                                                         |     |
|-----|---------------------------------------------------------|-----|
|     | A-2.2 Polar Form . . . . .                              | 429 |
|     | A-2.3 Conversion: Rectangular and Polar . . . . .       | 430 |
|     | A-2.4 Difficulty in Second or Third Quadrant . . . . .  | 431 |
| A-3 | Euler's Formula . . . . .                               | 431 |
|     | A-3.1 Inverse Euler Formulas . . . . .                  | 432 |
| A-4 | Algebraic Rules for Complex Numbers . . . . .           | 432 |
|     | A-4.1 Complex Number Exercises . . . . .                | 434 |
| A-5 | Geometric Views of Complex Operations . . . . .         | 434 |
|     | A-5.1 Geometric View of Addition . . . . .              | 435 |
|     | A-5.2 Geometric View of Subtraction . . . . .           | 436 |
|     | A-5.3 Geometric View of Multiplication . . . . .        | 437 |
|     | A-5.4 Geometric View of Division . . . . .              | 437 |
|     | A-5.5 Geometric View of the Inverse, $z^{-1}$ . . . . . | 437 |
|     | A-5.6 Geometric View of the Conjugate, $z^*$ . . . . .  | 438 |
| A-6 | Powers and Roots . . . . .                              | 438 |
|     | A-6.1 Roots of Unity . . . . .                          | 439 |
|     | A-6.1.1 Procedure for Finding Multiple Roots . . . . .  | 440 |
| A-7 | Summary and Links . . . . .                             | 441 |
| A-8 | Problems . . . . .                                      | 441 |

## **B    Programming in MATLAB 443**

|     |                                                     |     |
|-----|-----------------------------------------------------|-----|
| B-1 | MATLAB Help . . . . .                               | 444 |
| B-2 | Matrix Operations and Variables . . . . .           | 444 |
|     | B-2.1 The Colon Operator . . . . .                  | 445 |
|     | B-2.2 Matrix and Array Operations . . . . .         | 445 |
|     | B-2.2.1 A Review of Matrix Multiplication . . . . . | 445 |
|     | B-2.2.2 Pointwise Array Operations . . . . .        | 446 |
| B-3 | Plots and Graphics . . . . .                        | 446 |
|     | B-3.1 Figure Windows . . . . .                      | 447 |
|     | B-3.2 Multiple Plots . . . . .                      | 447 |
|     | B-3.3 Printing and Saving Graphics . . . . .        | 447 |
| B-4 | Programming Constructs . . . . .                    | 447 |
|     | B-4.1 MATLAB Built-in Functions . . . . .           | 448 |
|     | B-4.2 Program Flow . . . . .                        | 448 |
| B-5 | MATLAB Scripts . . . . .                            | 448 |

|       |                                          |     |
|-------|------------------------------------------|-----|
| B-6   | Writing a MATLAB Function . . . . .      | 448 |
| B-6.1 | Creating A Clip Function . . . . .       | 449 |
| B-6.2 | Debugging a MATLAB M-file . . . . .      | 451 |
| B-7   | Programming Tips . . . . .               | 451 |
| B-7.1 | Avoiding Loops . . . . .                 | 452 |
| B-7.2 | Repeating Rows or Columns . . . . .      | 452 |
| B-7.3 | Vectorizing Logical Operations . . . . . | 452 |
| B-7.4 | Creating an Impulse . . . . .            | 453 |
| B-7.5 | The Find Function . . . . .              | 453 |
| B-7.6 | Seek to Vectorize . . . . .              | 454 |
| B-7.7 | Programming Style . . . . .              | 454 |

## C Laboratory Projects 455

|         |                                                          |     |
|---------|----------------------------------------------------------|-----|
| C-1     | Introduction to MATLAB . . . . .                         | 457 |
| C-1.1   | Pre-Lab . . . . .                                        | 457 |
| C-1.1.1 | Overview . . . . .                                       | 457 |
| C-1.1.2 | Movies: MATLAB Tutorials . . . . .                       | 457 |
| C-1.1.3 | Getting Started . . . . .                                | 458 |
| C-1.2   | Warm-up . . . . .                                        | 458 |
| C-1.2.1 | MATLAB Array Indexing . . . . .                          | 459 |
| C-1.2.2 | MATLAB Script Files . . . . .                            | 459 |
| C-1.2.3 | MATLAB Sound (optional) . . . . .                        | 460 |
| C-1.3   | Laboratory: Manipulating Sinusoids with MATLAB . . . . . | 460 |
| C-1.3.1 | Theoretical Calculations . . . . .                       | 461 |
| C-1.3.2 | Complex Amplitude . . . . .                              | 461 |
| C-1.4   | Lab Review Questions . . . . .                           | 461 |
| C-2     | Encoding and Decoding Touch-Tone Signals . . . . .       | 463 |
| C-2.1   | Introduction . . . . .                                   | 463 |
| C-2.1.1 | Review . . . . .                                         | 463 |
| C-2.1.2 | Background: Telephone Touch-Tone Dialing . . . . .       | 463 |
| C-2.1.3 | DTMF Decoding . . . . .                                  | 464 |
| C-2.2   | Pre-Lab . . . . .                                        | 464 |
| C-2.2.1 | Signal Concatenation . . . . .                           | 464 |
| C-2.2.2 | Comment on Efficiency . . . . .                          | 465 |
| C-2.2.3 | Encoding from a Table . . . . .                          | 465 |
| C-2.2.4 | Overlay Plotting . . . . .                               | 465 |

|         |                                             |     |
|---------|---------------------------------------------|-----|
| C-2.3   | Warm-up: DTMF Synthesis . . . . .           | 465 |
| C-2.3.1 | DTMF Dial Function . . . . .                | 466 |
| C-2.3.2 | Simple Bandpass Filter Design . . . . .     | 467 |
| C-2.4   | Lab: DTMF Decoding . . . . .                | 468 |
| C-2.4.1 | Filter Bank Design: dtmfdesign.m . . . . .  | 468 |
| C-2.4.2 | A Scoring Function: dtmfscore.m . . . . .   | 469 |
| C-2.4.3 | DTMF Decode Function: dtmfscrun.m . . . . . | 470 |
| C-2.4.4 | Testing . . . . .                           | 471 |
| C-2.4.5 | Telephone Numbers . . . . .                 | 471 |
| C-2.4.6 | Demo . . . . .                              | 472 |
| C-3     | Two Convolution GUIs . . . . .              | 473 |
| C-3.1   | Introduction . . . . .                      | 473 |
| C-3.2   | Pre-Lab: Run the GUIs . . . . .             | 473 |
| C-3.2.1 | Discrete-Time Convolution Demo . . . . .    | 473 |
| C-3.2.2 | Continuous-Time Convolution Demo . . . . .  | 474 |
| C-3.3   | Warm-up: Run the GUIs . . . . .             | 475 |
| C-3.3.1 | Continuous-Time Convolution GUI . . . . .   | 475 |
| C-3.3.2 | Discrete Convolution GUI . . . . .          | 475 |
| C-3.4   | Lab Exercises . . . . .                     | 475 |
| C-3.4.1 | Continuous-Time Convolution . . . . .       | 475 |
| C-3.4.2 | Continuous-Time Convolution Again . . . . . | 476 |
| C-3.4.3 | Discrete-Time Convolution . . . . .         | 476 |

## D CD-ROM Demos 478

## Index 482