

M A T H E M A T I C A L
PERSPECTIVES ON
THEORETICAL PHYSICS

A Journey from Black Holes to Superstrings

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