

Mathematical and Computational Methods

Charles S. Tapiero

WILEY

## **Contents**

Preface				xiii
		Part	I: Finance and Risk Management	
Chapter 1	Potp	ourri		03
-	1.1	Introdu	action	03
	1.2	Theore	tical finance and decision making	05
	1.3	Insurar	nce and actuarial science	07
	1.4	Uncert	ainty and risk in finance	10
		1.4.1	Foreign exchange risk	10
		1.4.2	Currency risk	12
		1.4.3	Credit risk	12
		1.4.4	Other risks	13
	1.5	Financ	ial physics	15
	Sele	cted intr	oductory reading	16
Chapter 2	Mal	king Eco	onomic Decisions under Uncertainty	19
-	2.1	Decision	on makers and rationality	19
		2.1.1	The principles of rationality and bounded rationality	20
	2.2	Bayes	decision making	22
		2.2.1	Risk management	23
	2.3	Decisi	on criteria	26
		2.3.1	The expected value (or Bayes) criterion	26
		2.3.2	Principle of (Laplace) insufficient reason	27
		2.3.3	The minimax (maximin) criterion	28
		2.3.4	The maximax (minimin) criterion	28
		2.3.5	The minimax regret or Savage's regret criterion	28
	2.4	Decisi	on tables and scenario analysis	31
		2.4.1	The opportunity loss table	32
	2.5	EMV,	EOL, EPPI, EVPI	33
		2.5.1	The deterministic analysis	34
		2.5.2	The probabilistic analysis	34
	Sele	cted ref	erences and readings	38

viii CONTENTS

Chapter 3	Ex	pected [	<b>Itility</b>	39
	3.1	The c	oncept of utility	39
			Lotteries and utility functions	40
	3.2		y and risk behaviour	42
			Risk aversion	43
		3.2.2	Expected utility bounds	45
		3.2.3	Some utility functions	46
		3.2.4	Risk sharing	47
	3.3		nce, risk management and expected utility	48
		3.3.1		48
	3.4	Critiq	ues of expected utility theory	51
		3.4.1	Bernoulli, Buffon, Cramer and Feller	51
		3.4.2		52
	3,5	Expec	ted utility and finance	53
			Traditional valuation	54
			Individual investment and consumption	57
		3.5.3		59
		3.5.4	Portfolio and utility maximization in practice	61
		3.5.5	Capital markets and the CAPM again	63
		3.5.6	Stochastic discount factor, assets pricing	03
			and the Euler equation	65
	3.6	Inform	nation asymmetry	67
		3.6.1		68
		3.6.2	'The moral hazard problem'	69
		3.6.3	Examples of moral hazard	70
		3.6.4	Signalling and screening	72
		3.6.5		73
	Refe	rences a	and further reading	75
				13
Chapter 4	· · · · · · · · · · · · · · · · · · ·		79	
	4.1	Introdu	action	79
	4.2	Uncert	ainty, games of chance and martingales	81
	4.3	Uncert	ainty, random walks and stochastic processes	84
		4.3.1	The random walk	84
		4.3.2	Properties of stochastic processes	91
	4.4	Stocha	stic calculus	92
		4.4.1	Ito's Lemma	93
	4.5	Applic	ations of Ito's Lemma	94
		4.5.1	Applications	94
		4.5.2	Time discretization of continuous-time	
			finance models	96
		4.5.3	one of the order and man mignica	104
	References and further reading			108
	_			
Chapter 5	Derivatives Finance		111	

5.1 Equilibrium valuation and rational expectations

111

			CONTENTS	ix
	5.2	Financ	cial instruments	113
		5.2.1	Forward and futures contracts	114
		5.2.2	<u> </u>	116
	5.3		ng and institutions	119
			Hedging and hedge funds	120
		5.3.2		123
			Investor protection rules	125
	Refe	rences a	and additional reading	127
	Pa	rt II: M	Sathematical and Computational Finance	
Chapter 6	Opt	ions an	d Derivatives Finance Mathematics	131
	6.1		uction to call options valuation	131
		6.1.1	Option valuation and rational expectations	135
		6.1.2	1 <del>V</del>	137
		6.1.3		140
	6.2		rd and futures contracts	141
	6.3		neutral probabilities again	145
,		6.3.1	Rational expectations and optimal forecasts	146
	6.4		lack-Scholes options formula	147
			Options, their sensitivity and hedging parameters	151
			Option bounds and put-call parity	152
			American put options	154
	Refe	rences	and additional reading	157
Chapter 7	Opt	Options and Practice		
	7.1	Introd		161
	7.2		ged options	163
	7.3		ound options and stock options	165
			Warrants	168
			Other options	169
	7.4	_	ns and practice	171
		7.4.1	3	172
		7.4.2	· · · · · · · · · · · · · · · · · ·	
			share	176
		7.4.3	1 1 5 5 5	100
		244	put and a stock	177
1.0		7.4.4	1 0	178
			Straddle and strangle strategies	179
			Strip and strap strategies	180
		7.4.7		181 181
	7 5	7.4.8	Dynamic strategies and the Greeks ing time strategies*	181
	7.5	5.0pp 7.5.1	Stopping time sell and buy strategies	184
	7.6		fic application areas	195
	7.0	- specil	ne apprication areas	173

	7.7	Option	n misses	197
	References and additional reading			204
	Appendix: First passage time*			
Chapter 8	Fixed Income, Bonds and Interest Rates			211
•	8.1		and yield curve mathematics	211
		8.1.1	•	213
			Coupon-bearing bonds	215
			Net present values (NPV)	217
			Duration and convexity	218
	8.2		and forward rates	222
	8.3 Default bonds and risky debt			224
	8.4 Rated bonds and default			230
		8.4.1	A Markov chain and rating	233
		8.4.2	Bond sensitivity to rates - duration	235
			Pricing rated bonds and the term structure	
			risk-free rates*	239
		8.4.4	Valuation of default-prone rated bonds*	244
	8.5	Intere	st-rate processes, yields and bond valuation*	251
		8.5.1	The Vasicek interest-rate model	254
		8.5.2	Stochastic volatility interest-rate models	258
		8.5.3	Term structure and interest rates	259
	8.6	Option	ns on bonds*	260
-		8.6.1	Convertible bonds	261
		8.6.2	Caps, floors, collars and range notes	262
		8.6.3	Swaps	262
	References and additional reading			264
	Mathematical appendix			267
		A.1:	Term structure and interest rates	267
		A.2:	Options on bonds	268
Chapter 9	Incomplete Markets and Stochastic Volatility 27			
	9.1 Volatility defined		271	
	9.2	Memo	ry and volatility	273
	9.3	Volati	lity, equilibrium and incomplete markets	275
		9.3.1	Incomplete markets	276
	9.4	Proces	ss variance and volatility	278
	9.5	Implicit volatility and the volatility smile		281
	9.6	Stocha	astic volatility models	282
		9.6.1	Stochastic volatility binomial models*	282
		9.6.2	Continuous-time volatility models	00
	9.7	Equilibrium, SDF and the Euler equations*		
	9.8	Select	ed Topics*	295
		9.8.1	The Hull and White model and stochastic	
			volatility	296
		9.8.2	Options and jump processes	297

		CONTENTS	xi		
9.9	The range process and volatility				
Refer		d additional reading	301		
		velopment for the Hull and White model (1987)*	305		
Value	e at Risk	and Risk Management	309		
10.1					
10.2	VaR de	finitions and applications	311		
10.3	VaR sta	tistics	315		
	10.3.1	The historical VaR approach	315		
	10.3.2	The analytic variance-covariance approach	315		
		VaR and extreme statistics	316		
	10.3.4	Copulae and portfolio VaR measurement	318		
	10 3 5	Multivariate risk functions and the			

320

324 324

324 326

327

329

333

principle of maximum entropy

VaR and portfolio risk efficiency with

10.3.6 Monte Carlo simulation and VaR

normal returns

10.4.2 VaR and regret References and additional reading

9.9

10.4

VaR efficiency 10.4.1

Chapter 10

Author Index

Subject Index