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Complex Analysis

$$\int_R \nabla \cdot \vec{F} dV = \int_{\partial R} \vec{F} \cdot \vec{n} d\sigma \longleftrightarrow \int_R dw = \int_{\partial R} w$$

$$\sim (P \cdot Q) = \sim P \vee \sim Q, \sim (P \vee Q) = \sim P \cdot \sim Q$$

$$|\langle \chi, \gamma \rangle| \leq \| \chi \| \| \gamma \|$$

$$\delta_j = \frac{1}{|G|} \sum_{g \in G} x(g) \overline{x_j(g)} = \frac{1}{|G|} \sum_{j=1}^r k_j x(g_j) \overline{x_j(g_j)}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



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$$\int_a^b f(t) dt = F(b) - F(a)$$



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