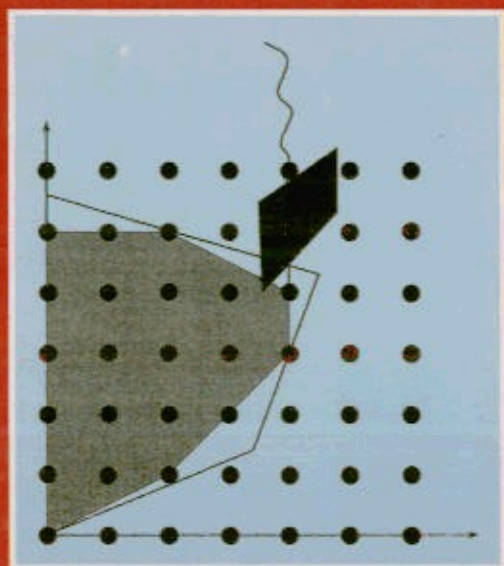


CAMBRIDGE TEXTS
IN APPLIED
MATHEMATICS

A First Course in Combinatorial Optimization



JON LEE

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