

Design of Experiments.

Statistical Principles of Research Design and Analysis

Second Edition

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Research Design Principles

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