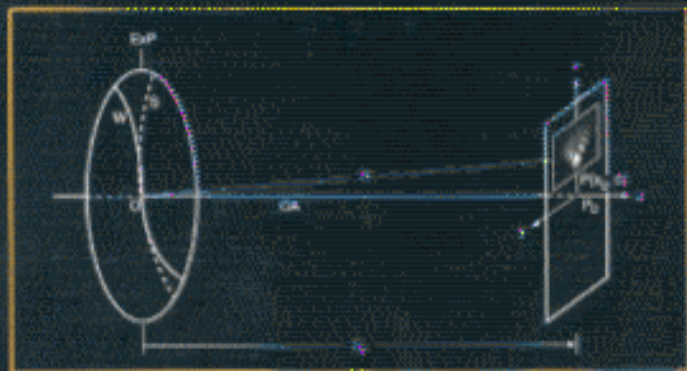


OPTICAL IMAGING AND ABERRATIONS

PART II

WAVE DIFFRACTION OPTICS



VIRENDRA N. MAHAJAN

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