

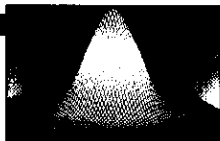


PROBABILITY and RANDOM PROCESSES

With Applications to
Signal Processing and
Communications

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