

A Course in  
**Modern  
Mathematical  
Physics** Groups,  
Hilbert Space  
and Differential  
Geometry

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CAMBRIDGE

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