



GEORGE ELLIS

# CONTROL SYSTEM DESIGN GUIDE

USING YOUR COMPUTER TO UNDERSTAND  
AND DIAGNOSE FEEDBACK CONTROLLERS

THIRD EDITION

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