

$$\pi(\mathbf{y}) = \begin{bmatrix} E(y_1) \\ E(y_2) \\ \vdots \\ E(y_n) \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1 \\ \beta_0 + \beta_1 x_2 \\ \vdots \\ \beta_0 + \beta_1 x_n \end{bmatrix} = \mathbf{X}\boldsymbol{\beta}$$

# Biostatistics for Animal Science



M Kaps and W Lamberson



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